Clifford Tetrads, Null Zig Zags, and Quantum Gravity*

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Abstract

Quantum gravity has been so elusive because we have tried to approach it by two paths which can never meet: standard quantum field theory and general relativity. These contradict each other, not only in superdense regimes, but also in the *vacuum*, where the divergent zero-point energy would roll up space to a point. The solution is to build in a regular, but topologically nontrivial distribution of *vacuum spinor fields* right from the start. This opens up a straight road to quantum gravity, which we map out here.

The gateway is covariance under the *complexified Clifford algebra* of our space-time manifold $\mathbb{M}$, and its spinor representations, which Sachs dubbed the *Einstein group, E*. The 16 generators of E transformations obey both the *Lie algebra* of *Spin*^{c}*-4*, and the Clifford (SUSY) algebra of $\mathbb{M}$. We derive Einstein's field equations from the simplest E-invariant Lagrangian density, $\mathcal{L}_g$. $\mathcal{L}_g$ contains effective *electroweak* and *gravitostrong field* actions, as well as Dirac actions for the matter spinors. On microscales the massive Dirac propagator resolves into a sum over *null zig-zags*. On macroscales, we see the *energy-momentum current,* $*T$, and the resulting Einstein curvature, G.

For massive particles, $*T$ flows in the "cosmic time" direction—*centrifugally* in an expanding universe. Neighboring centrifugal currents of $*T$ present *opposite* radiotemporal vorticities G_{or} to the boundaries of each others' worldtubes, so they *advect,* i.e. *attract,* as we show here by integrating $\mathcal{L}_g$ by parts in the *spinfluid* regime. This boundary integral not only explains *why* stress-energy is the source for gravitational curvature,

*This paper gives the derivations of the results I reported at the PIMS conference entitled "Brane World and Supersymmetry" in July, 2002 at Vancouver, B.C. It also contains new results on spin-gravity coupling, on how a topologically-nontrivial distribution of vacuum spinors removes singularities and divergences, and how the amplitude of the vacuum spinors determines the gravitational constant and the rate of cosmic expansion.

but also gives a *value* for the gravitational constant, $\kappa(T)$, that couples them. κ turns out to depend on the dilation factor $T = y^0$, which enters kinematically as "imaginary time": the logradius of our expanding Friedmann 3-brane.

On the microscopic scale, *quantum gravity* appears as the *statistical mechanics* of the *null zig-zag* rays of spinor fields in imaginary time T. Our unified field/particle action $\mathcal{L}_g$ also contains new couplings of gravitomagnetic fields to strong fields and weak potentials. These *predict new physical phenomena: Axial jets* of nuclear decay products emitted with *left helicity* along the axis of a massive, spinning body.

1 Overview: Vacuum Energy and Inertial Mass

Quantum Field Theory (QFT) is *incompatible* with General Relativity (GR), not only at small scales or high densities, but in the *vacuum!* The problem is that the divergent QFT vacuum energy would produce enough spacetime curvature to roll up our space to a point. The *solution* is a fundamental theory from which both QFT and GR derive, in different regimes. We show here how quantum gravity emerges naturally from such a theory: the Nonlinear Multispinor (NM) model [1], [2].

In this model, the particles emerge [3], [4] through *dynamical symmetry breaking* of a topologically nontrivial "vacuum" solution. Their interactions are mediated by the phase perturbations they leave in the residual vacuum around them.

We may summarize the role of the vacuum spinors in producing inertial and gravitational mass [2] like this:

> *Nonlinear interaction of opposite-chirality components through the vacuum spinor fields creates the inertial mass of a bispinor particle. It also produces the gravitational interaction between particles.*

Inertial mass from global interaction—and its corollary, gravitation, are *results* we *derive* from the NM model. We thus supply a *mechanism* that embodies *Mach's principle* (inertia from interaction with the "distant masses"), and Einstein's principle (inertial mass equals gravitational mass). Let us start with the "big picture", and then zoom in to the microscopic level.

The new "precision cosmology" [5] has suddenly given us a clear view of the universe in which we live. We live on a 3-*brane* (hypersurface) $S_3(a)$ whose (local) radius $a(x)$ is expanding with intrinsic clocktime: Minkowsky time, $t \equiv x^0$; the arclength travelled by a photon, projected to $\mathbb{S}_3(a_\#)$ [6]. We take S_3 to be a *closed* hypersurface—a deformed, nonuniformly expanding 3-sphere.

If we could rise above our expanding spatial hypersurface S_3 and look down on it from the direction $T = y^0$ of cosmic expansion, we would see a patchwork quilt of matter concentrations connected by the fabric we call "space". It is becoming clear that space is not empty. It is filled with "dark energy", which includes the quantum mechanical *vacuum energy* and comprises over 70% of the

energy in the universe [7]. On the macroscopic scale, this is what produces the cosmological constant term that brings the average energy (almost exactly) to the critical value needed to just close the universe.

Of what is this vacuum energy made? Before we get a microscopic view of the vacuum, let us first zoom in on some of the matter concentrations that it separates—starting with a single electron. The worldtube, B_4, of an electron (or *any* massive particle at rest) runs in the *cosmic time* direction, T, orthogonal to the 3 spatial directions that span our *spatial* 3*-brane,* $S_3(T)$.

The chiral components ξ_- and η_- of e_- are *spinors:* lightlike waves of internal $u(1) \oplus su(2)$ phase with *definite helicity* (spin in the direction of propagation). Since ξ_- and η_- have *opposite helicities* but the *same spins,* they must be *counterpropagating—radially outward and inward for an electron at rest.* Since the rest frame of a massive particle drifts *slowly* outward with cosmic expansion, there must be *almost* as many inward ("backward" in T) as outward ("forward") lightlike ray segments within its worldtube [8].

At the worldtube boundary ∂B_4, zigs are scattered into zags and vice versa by nonlinear interactions with the vacuum spinors [2]. It is these *mass scatterings* [9] that keep the lightlike spinors of a massive bispinor particle confined to a timelike worldtube. Energy-momentum, angular momentum, and all internal quantum numbers are conserved at each mass scattering, where 4 Lie-algebra phases of incoming and outgoing spinors combine to give a *scalar* contribution to the action.

Mass scatterings, or *Spin*^{c}*-4 resonances,* are the multispinor analog of the *Bragg resonances* (4-wave mixing) [10] which produce the self-trapping that leads to soliton formation in nonlinear optics. The same nonlinearity gives the *soliton interactions.* These are most easily calculated for conservative fields by integrating the Lagrangian density by parts over the boundary of the worldtube of an accelerated particle. Matching the *inner form* of the boundary integral in the localized soliton fields to the *outer form* in the vacuum fields, as perturbed by source distributions, gives the curvature of the particle's worldtube [11].

We derive Einstein's field equations here by integrating a topological Lagrangian by parts. The crucial step is to recognize that Einstein curvature G and energy-momentum $*T$ are different expressions for the *same* flux, *the spinfluid current* 3 *form.* Inside the worldtube, B_4, of a particle, this takes the form of the stress-energy-momentum tensor [12], $*T$. This is Noether current under displacement of the *boundary* ∂B^4 of the worldtube. In the outer region, the spinfluid current takes the form of the *Einstein curvature* [13], G. By matching the inner and outer forms on the *moving boundary* $\partial B_4(t)$, we obtain Einstein's field equations.

We obtain the quantum mechanical form of this boundary integral by focusing on a patch of boundary at a microscopic scale, where the matter current is resolved into a sum over null zig-zags [9]; mass-scatterings with the vacuum fields, as perturbed by source distributions. Analyticity conditions convert the *statistical mechanics* of mass scatterings in imaginary time or *logradius* $T \equiv y^0$ to a quantum theory of the mutual attraction of matter currents: *quantum gravity.*

2 Spinfluid Flow: The Dilation-Boost Current

Conserved currents spring from *invariances* under the group of spin isometries of our spacetime manifold $\mathbb{M}$: the *Einstein* (E) *group* [14]. Passive Einstein (E_P) transformations relate the same physical state ψ in different frames of reference. Active Einstein (E_A) transformations change ψ in a way that can be undone by *local, path dependent* E_P transformations. *Spin curvatures,* or *fields,* are the obstruction to "combing" ψ to covariant constancy by *any* (path-independent) E_P transformation.

Cosmic expansion and boost-covariance demand that E be *nonunitary*—i.e. have *Hermitian* (H) as well as *anti-Hermitian* (aH) generators. The *energy-momentum* current, $*T$, is the *Noether current of the spinor fields under space-time translations* [12]. In an expanding universe [6], the net *dilation flow* is *centrifugally outward,* in the direction T of cosmic expansion. Cosmic time $T \equiv y^0$ enters kinematically [6], [9] as the *imaginary* part of a *complex time* variable $z^0 \equiv x^0 + iy^0$, where $x^0 \equiv t$ is *Minkowsky time.* y^0 transforms like an *energy.* The imaginary parts y^j of $z^j \equiv x^j + iy^j$ transform like 3-momenta. The *dilation current* is the *energy density*, the boost current is the momentum density. The 4 complex variables z^α are coordinates on the *position-momentum phase space* [2], [6]:

$$\mathbb{M} \subset T^*\mathbb{M} \subset \mathbb{C}_4;$$

the base space for the bundle of *vacuum* $\oplus$ *matter spinors.* In the *spinfluid regime*, the dilation-boost flow $y^\alpha(x)$ is a function of Minkowsky-space position $x \equiv (x^0, x^1, x^2, x^3) \in \mathbb{M}_\#$.

The spin isometry group of $T^*\mathbb{M}$—the globalization of the Poincaré group—is the *Einstein group* [14]

$$E \sim Spin^c\text{-}4 \sim Gl\,(2,\mathbb{C})_L \times Gl\,(2,\mathbb{C})_R\,,$$

the complexification of $(Spin\ 4) \times U(1)$—a 16-parameter Lie group. E covers the *conformal group*[1] with one extra $U(1)$—or *electromagnetic* parameter. The parameters $x^\alpha \in [0, 4\pi a_\#]$ that multiply the aH generators are canonical $u(1) \times su(2,\mathbb{C})$ *translation* parameters on $\mathbb{M}_\# \equiv \mathbb{S}_1 \times \mathbb{S}_3(a_\#)$, Penrose's [9] conformal compactification of Minkowsky space. $\mathbb{S}_3(a_\#)$ is a reference 3-sphere of radius $a_\#$. The fundamental length unit $a_\#$ turns out [6], [15] to be the *equilibrium radius* of the Friedmann solution [6] (see Appendix).

We treat the *vacuum* $\widehat{\mathbb{M}} \equiv \mathbb{M} \backslash \cup D_J$ outside the worldtubes of massive particles as a *spinfluid:* an inhomogeneous (but regular) distribution of 4 path-dependent spinors $\boldsymbol{\psi}_I(x)$ and 4 cospinors $\boldsymbol{\psi}^I(x)$, governed by a Lagrangian density, $\mathcal{L}_g$. $\mathcal{L}_g$ is invariant under the Lie group of passive (E_P) changes in the background spin and spacetime frames, augmented by the discrete involutions P (space-reversal), T (time reversal), and C (charge reversal). A PTC invariant inner product $\psi^I_\mp \psi^\pm_I$ is made between spinors $\psi^\pm_I$ and *cospinors* $\psi^I_\pm$ of *Opposite*

[1] The chiral $GL\,(2,\mathbb{C})$ presentation turns out to be better suited than the twistor presentation for unifying Dirac mass with gravitation—a fact first pointed out to me by Jaime Keller [20].

Charge ($u(1)$ phase shift with T), *Parity* ($su(2)$ phase advance along rays), and *Temorality* (inward, i.e. forward, or outward, i.e. backward, propagation in T).

A good way to describe a physical state is by the *active local* (E_A) transformation, which creates this state from the vacuum. Suppose that each spinor field may be created from the *vacuum* distribution $\widehat{\boldsymbol{\psi}}_{\pm}$ by (a path-dependent) active-local Einstein (E_A) transformation [6], [16]. These act on the *basis spinors* to create the *moving spin frames* $g_I(x)$ written as $gl(2,\mathbb{C})$ matrices. Each spinor $\psi_I^{\pm}$ and cospinor $\psi_{\mp}^{I}$ is expressed as a linear combination of the two ("spin-up" and "spin-down") basis spinors in its moving spin frame, with coefficients given by the column or row *spin vectors* $\boldsymbol{\psi}_I^{\pm}(x)$ or $\boldsymbol{\psi}_{\pm}^{I}(x)$:

$$\begin{gathered}\psi_I^{\pm}(x) = \exp\left(\tfrac{i}{2}\zeta_I^{\alpha\pm}(x)\,\sigma_\alpha\right)\boldsymbol{\psi}_I^{\pm} \equiv g_I^{\pm}(x)\,\boldsymbol{\psi}_I^{\pm},\\ \psi_{\pm}^{I}(x) = \boldsymbol{\psi}_{\pm}^{I}\exp\left(\tfrac{i}{2}\zeta^{I\alpha\pm}(x)\,\overline{\sigma}_\alpha\right) \equiv \boldsymbol{\psi}_{\pm}^{I} g_{\pm}^{I}(x)\,;\\ \alpha = (0,1,2,3)\,,\end{gathered} \tag{1}$$

where $\overline{\sigma}_\alpha \sim (\sigma_0, -\sigma_1, -\sigma_2, -\sigma_3)$ is the Lie algebra dual to σ_α under the *Clifford product*

$$\sigma_\alpha\overline{\sigma}_\beta + \sigma_\beta\overline{\sigma}_\alpha = 2\eta_{\alpha\beta}$$

for Minkowsky space. The $\pm$ signs indicate the *charge* ($u(1)$ phase shift) of the field. (We shall sometimes drop the charge scripts below.) Spinors and cospinors must be *varied independently* in the Lagrangian, which must contain *both* to be a *scalar* under E_P.

In the *geometrical-optics* (g.o.) regime, each spinor has a *complex*, nonsingular (but perhaps path-dependent) phase (1) with (perhaps inexact) $gl(2,\mathbb{C})$ phase differential $\mathbf{d}\zeta_I^{\alpha}(x)$. In g.o. solutions, the cospinor turns out to be [2] the PTC-reversed version of the spinor, with the *opposite* $gl(2,\mathbb{C})$ phase shift.

When you differentiate a spinor, you must also differentiate its moving spin frame:

$$\begin{gathered}\mathbf{d}\psi_I(x) = \mathbf{d}\left(g_I(x)\,\boldsymbol{\psi}_I(x)\right) = g_I\mathbf{d}\boldsymbol{\psi}_I + \mathbf{d}g_I\boldsymbol{\psi}_I\\ \equiv g_I\left[\mathbf{d} + g_I^{-1}\mathbf{d}g_I\right]\boldsymbol{\psi}_I \equiv g_I\left[\mathbf{d} + \Omega_I(x)\right]\boldsymbol{\psi}_I(x) \equiv g_I\boldsymbol{\nabla}\boldsymbol{\psi}_I.\end{gathered} \tag{2}$$

The *spin connections* (vector potentials)

$$g_I^{-1}\mathbf{d}g_I = \frac{i}{2}\mathbf{d}\zeta_I^{\alpha}(x)\,\sigma_\alpha = \frac{i}{2}\left[\mathbf{d}\theta_I^{\alpha}(x) + i\mathbf{d}\varphi_I^{\alpha}\right]\sigma_\alpha \equiv \Omega_I(x) \tag{3}$$

thus enter as $gl(2,\mathbb{C})$-valued 1 forms into the *covariant derivatives* of each spinor field:

$$\nabla_\beta\boldsymbol{\psi}_I \equiv \left(\partial_\beta + \Omega_{I\beta}(x)\right)\boldsymbol{\psi}_I.$$

The $\Omega_I(x)$ record the (perhaps path-dependent) phase shift of each moving spin frame $g_I(x)$, in any direction at point $x \in \mathbb{M}$ due to local sources—and of the global (vacuum) distribution.

The simplest way to guaranty covariance of the wave equations for spinor, or *spin-tensor* fields is to write the spacetime position vector $q(x)$ from the origin,

$x=0$, to the spacetime position point q, as the *position quaternion*

$$q(x) \equiv a\sigma_0 \exp\left[\tfrac{i}{a_\#} x^\alpha \sigma_\alpha\right] \equiv a^0(x)\,\sigma_0 + i a^j(x)\,\sigma_j \in \mathbb{M}_\#: \\ \left(a^0\right)^2 + \left(a^j\right)^2 = a \qquad (\text{sum on } j=1,2,3). \tag{4}$$

The point has an *embedded radius*

$$a(x) \equiv \exp\left[\tfrac{1}{a_\#} y^0(x)\right] a_\# \equiv \gamma(x)\, a_\#; \\ \gamma(x) \equiv \tfrac{a(x)}{a_\#} \tag{5}$$

is the (local) *scale factor.*

For the stationery, homogeneous vacuum $\mathbb{M}_\# \equiv \mathbb{S}_1 \times \mathbb{S}_3(a_\#)$, the *left-invariant spin connections* are the left-invariant Maurer-Cartan 1 forms that derive from *right* action on $q(x)$ by the four canonical maps of $\mathbb{M}_\#$ onto $U(1) \times SU(2)$ (and *vice versa*):

$$g^\pm(x) \equiv \exp \frac{i}{2a_\#} x^\alpha \sigma^\pm_\alpha; \qquad \overline{g}^\pm(x) \equiv \exp \frac{i}{2a_\#} x^\alpha \overline{\sigma}^\pm_\alpha; \tag{6}$$

where $\sigma_\alpha \equiv (\pm\sigma_0, \sigma_1, \sigma_2, \sigma_3)$ and $\bar{\sigma}^\pm_\alpha = (\pm\sigma_0, -\sigma_1, -\sigma_2, -\sigma_3)$.

On our *Friedmann vacuum* $\widehat{\mathbb{M}}$, with time-dependent scale factor $\frac{y^0(t)}{a_\#} = \gamma(t)$, the left-invariant spin connections are

$$\hat{\Omega}^\pm_L \equiv g^L_\mp \mathbf{d} g^\pm_L = \tfrac{i}{2a_\#} \sigma^\pm_\alpha e^\alpha - \tfrac{1}{2}\dot{\gamma}\sigma_0 e^0 \\ \hat{\Omega}^\pm_R \equiv g^R_\mp \mathbf{d} g^\pm_R = \tfrac{i}{2a_\#} \bar{\sigma}^\pm_\alpha e^\alpha - \tfrac{1}{2}\dot{\gamma}\sigma_0 e^0, \tag{7}$$

where

$$g^\pm_L \equiv g^\pm \exp\left(-\tfrac{y^0}{2a_\#}\right) \equiv \left(g^L_\pm\right)^{-1}, \\ g^\pm_R \equiv \overline{g}^\pm \exp\left(-\tfrac{y^0}{2a_\#}\right) \equiv \left(g^R_\pm\right)^{-1}.$$

The right-invariant spin connections are

$$\Omega^L_\pm \equiv \left(\mathbf{d} g^\pm_L\right) g^L_\mp; \\ \Omega^R_\pm \equiv \left(\mathbf{d} g^\pm_R\right) g^R_\mp. \tag{8}$$

Effective spin connections (3) are formed by *differentials* of each spinor field, when multiplied by its PTC conjugate spinor:

$$\psi^I \mathbf{d}\psi_I = \boldsymbol{\psi}^I g_I \mathbf{d}\left(g^I \boldsymbol{\psi}_I\right) = \boldsymbol{\psi}^I \Omega_I \boldsymbol{\psi}_I; \\ \Omega_I \equiv \tfrac{i}{2}\mathbf{d}\zeta_I \equiv \tfrac{i}{2}\mathbf{d}\zeta^\alpha_I \sigma_\alpha = \tfrac{i}{2}\mathbf{d}\theta^\alpha_I - \tfrac{1}{2}\mathbf{d}\varphi^\alpha_I. \tag{9}$$

The *anti-Hermitian* (aH) parts $\frac{i}{2}\mathbf{d}\theta^\alpha\sigma_\alpha$ of the spin connections are the $u(1) \times su(2)$ *electroweak vector potentials.* The *Hermitian* (H) parts $-\frac{1}{2}\mathbf{d}\varphi^\alpha\sigma_\alpha$ are the *gravitational* potentials. These measure the local *dilation-boost flow* $\mathbf{d}\varphi^\alpha(x)$ of the spinfluid [6], [16]. The path dependences, or *holonomies,*

$$g^I \mathbf{dd} g_I \equiv K_I = \mathbf{d}\Omega_I + \Omega_I \wedge \Omega_I, \tag{10}$$

of the spin connections are the *spin curvatures,* or *fields.*

The dilation/boost flow acts on the position quaternion, (4) to produce the *vacuum energy-momentum* distribution

$$\exp\left[\frac{i}{a_{\#}}z^{\alpha}\left(x\right)\sigma_{\alpha}\right]\equiv\left(iq+p\right)\left(x\right)\in\mathbb{CM}_{\#}\subset T^{*}\mathbb{M}_{\#}.$$

This assigns a *position/momentum quaternion* $(iq+p)(x)$ in the *phase space* $T^{*}\mathbb{M}_{\#}$ to each regular point $x\in\mathbb{M}_{\#}$ on the base space. The *complex structure* on the complex quaternionic phase space $\mathbb{CM}_{\#}$ gives rise to the *symplectic structure* of particle orbits on $T^{*}\mathbb{M}_{\#}$ [17].

The dilation parameter

$$\varphi^{0}\left(x\right)=a_{\#}^{-1}y^{0}\left(x\right)$$

encodes local concentrations of rest energy, or *mass.* Unlike the boost current $\mathbf{d}\varphi^{j}(x)$, the local energy current $\mathbf{d}\varphi^{0}(x)$ is not directly visible to us as dwellers in a spacelike (constant y^{0}) cross section, because it denotes the component p^{0} of the 4-momentum flux *normal* to our expanding space $S_{3}(t)$.

However, if the phase flow

$$\mathbf{d}\varsigma^{\alpha}\equiv\frac{\partial\zeta^{\alpha}}{\partial z^{\beta}}\mathbf{d}z^{\beta}+\frac{\partial\zeta^{\alpha}}{\partial\bar{z}^{\beta}}\mathbf{d}\bar{z}^{\beta}\tag{11}$$

were *analytic,* it would obey the Cauchy-Riemann equations:

$$\frac{\partial\varsigma^{\alpha}}{\partial\bar{z}^{\beta}}=0\Longrightarrow-\frac{\partial\theta^{\alpha}}{\partial x^{\beta}}=\frac{\partial\varphi^{\alpha}}{\partial y^{\beta}};\quad\frac{\partial\theta^{\alpha}}{\partial y^{\beta}}=\frac{\partial\varphi^{\alpha}}{\partial x^{\beta}}.\tag{12}$$

We could then detect the dilation current, or *rest energy,* by the *frequency*

$$-\frac{\partial\theta^{0}}{\partial x^{0}}=\frac{\partial\varphi^{0}}{\partial y^{0}}=\frac{m}{\hbar}\tag{13}$$

of the matter wave, ψ. But *energy* is the Noether charge under Minkowsky-time translation. For the Dirac Lagrangian, $\mathcal{L}_{D}$ [2], [16], this turns out to be Planck's constant times the frequency (13), which we *can* detect:

$$\int_{B_{3}}\frac{\partial\mathcal{L}_{D}}{\partial\left(\partial_{0}\psi_{I}\right)}\left[\frac{\partial\psi_{I}}{\partial x^{0}}\right]d^{3}v=\hbar\int_{B_{3}}-\left(\frac{\partial\theta^{0}\left(x\right)}{\partial x^{0}}\right)e^{1}\wedge e^{2}\wedge e^{3}.\tag{14}$$

3 Dirac Spinors and Clifford Vectors

Covariance of the Dirac equations on a curved, expanding spacetime $\mathbb{M}$ [18] rests on the local *spinorization maps*

$$\begin{aligned}S&\equiv q_{\alpha}\left(x\right)E^{\alpha}\left(x\right):E_{\beta}\left(x\right)\longrightarrow q_{\beta}\left(x\right);\\\bar{S}&\equiv\bar{q}_{\alpha}\left(x\right)E^{\alpha}\left(x\right):E_{\beta}\left(x\right)\longrightarrow\bar{q}_{\beta}\left(x\right).\end{aligned}\tag{15}$$

These assign the fields; $q_\beta(x) \in gl(2\mathbb{C})_L$, $\bar{q}_\beta(x) \in gl(2\mathbb{C})_R$, of *moving tetrads*—complex quaternions—to each intrinsic spacetime increment $E_\alpha(x) \in T\mathbb{M}$. The tetrads obey the *Clifford algebra* of $\mathbb{M}$:

$$[q_\alpha \bar{q}_\beta + q_\beta \bar{q}_\alpha](x) = 2g_{\alpha\beta}(x)\,\sigma_0, \tag{16}$$

where $g_{\alpha\beta}(x)$ is the *metric tensor*, which gives the *scalar products* of Clifford tetrads, pairwise. The overbar denotes *quaternionic* conjugation—space (P) reversal.

The Clifford tetrads are sums of *null tetrads:* tensor products of some fundamental L- and R-chirality physical fields—the inertial spinor fields, or *vacuum spinors* (VS) [9], [19]: the covariantly constant (null) modes of the Dirac operators. The local tetrads are *defined* as $gl(2,\mathbb{C})$ matrices, with respect to the vacuum spinors as bases:

$$q_\alpha(x) = \sigma_\alpha^{\ A\dot{B}} \ell_A(x) \otimes r^T_{\dot{B}}(x), \qquad \bar{q}_\alpha = \sigma_\alpha^{\ \dot{U}V} r_{\dot{U}}(x) \otimes \ell^T_V(x), \tag{17}$$

(sum on A, $\dot{B} = 1, 2$).

On an expanding, *homogeneous* Friedmann 3-sphere $(T, \mathbb{S}_3(T))$ with dilation parameter $y^0 \equiv T$, the normalized vacuum spinors and cospinors are

$$\begin{aligned}
\ell^\pm &\equiv \gamma^{-\frac{1}{2}} \widehat{\ell}^\pm \equiv \left[\ell_1^+, \ell_2^-\right](x) = \sigma_0 \exp \tfrac{i}{2a_\#}\left[\left(\pm x^0 + iy^0\right)\sigma_0 + x^j \sigma_j\right], \\
r^\mp &\equiv \gamma^{-\frac{1}{2}} \widehat{r}^\mp \equiv \left[r_{\dot{1}}^-, r_{\dot{2}}^+\right](x) = \sigma_0 \exp \tfrac{i}{2a_\#}\left[\left(\mp x^0 + iy^0\right)\sigma_0 + x^j \overline{\sigma}_j\right], \\
r_\pm &\equiv \gamma^{-\frac{1}{2}} \widehat{r}_\pm \equiv \left[r_+^{\dot{1}}, r_-^{\dot{2}}\right]^T(x) = \exp \tfrac{i}{2a_\#}\left[\left(\pm x^0 + iy^0\right)\sigma_0 + x^j \overline{\sigma}_j\right]\sigma_0, \\
\ell_\mp &\equiv \gamma^{-\frac{1}{2}} \widehat{\ell}_\mp \equiv \left[\ell_-^1, \ell_+^2\right]^T(x) = \exp \tfrac{i}{2a_\#}\left[\left(\mp x^0 + iy^0\right)\sigma_0 + x^j \sigma_j\right]\sigma_0.
\end{aligned} \tag{18}$$

The $(+)$ charge index indicates propagation of the $u(1)$ phase *outward* (in the direction of cosmic expansion), $\frac{\partial\theta^0}{\partial T} > 0$; a $(-)$ charge index indicates *inward* propagation. For the vacuum spinors, charge is coupled to spin [2]. We have indicated their charges in the opposite position to their spin indices in (18); then suppressed spin indices by writing the moving frames for spinors and cospinors as $GL(2,\mathbb{C})$ matrices columnwise and row-wise respectively. PTC-reversal of a spinor produces the *dual* cospinor, an *equivalent* representation of the E group [2]. Thus there are **4** fundamental E representations:

$$\begin{array}{ll}
\ell^+ \sim r_- & r^+ \sim \ell_- \\
\ell^- \sim r_+ & r^- \sim \ell_+ .
\end{array}$$

The vacuum spinors in (18) have *conformal weights* $\gamma^{-\frac{1}{2}}$, where γ is the scale factor (5) of our expanding spatial hypersurface $S_3(T)$. This assures that the covariant tetrads (17), metric tensor (16), and the intrinsic volume element all scale properly with γ:

$$\begin{gathered}
|q_\alpha| \sim \gamma^{-1} \sim \left|\overline{q}_\beta\right| : g_{\alpha\beta} \sim \gamma^{-2} \\
|E^\alpha| \sim \gamma^{+1} : g^{\alpha\beta} \sim \gamma^{+2}; \\
e^0 \wedge e^1 \wedge e^2 \wedge e^3 = |g|^{\frac{1}{2}} E^0 \wedge E^1 \wedge E^2 \wedge E^3 = \gamma^{-4} E^0 \wedge E^1 \wedge E^2 \wedge E^3.
\end{gathered} \tag{19}$$

The vacuum intensity $\ell^{\pm}r_{\mp}$ scales as γ^{-1} with cosmic expansion; the intensity of the vacuum spinors at $T=0$, on $\mathbb{M}_{\#}$, has been normalized to 1 (c.f. [2]). The *vacuum amplitude* scales as $\gamma^{-\frac{1}{2}} = \exp\left[-\frac{T}{2a_{\#}}\right]$, so the vacuum energy density γ^{-4} integrates to 1, a *constant*, in the dilated frame (19).

We denote the bundle of vacuum spinors over $\mathbb{M}_{\#}$ as perturbed, pathwise, by local sources as

$$\begin{aligned} \ell^{\pm}(x) &\equiv \gamma^{-\frac{1}{2}}\widehat{\ell}^{\pm}\exp\tfrac{i}{2}\left[\theta_L^{\alpha\pm}(x) + i\varphi_L^{\alpha\pm}(x)\,\sigma_\alpha\right] \\ r^{\pm}(x) &\equiv \gamma^{-\frac{1}{2}}\widehat{r}^{\pm}\exp\tfrac{i}{2}\left[\theta_R^{\alpha\pm}(x) + i\varphi_R^{\alpha\pm}(x)\,\overline{\sigma}_\alpha\right] \end{aligned} \tag{20}$$

Tensor products of vacuum spinors make the *null tetrads.* Sums and differences of these make the (*spin*-1) tetrads (17). Products of the tetrads make the (*spin*-2) metric tensor (16).

It is a remarkable fact [6] that (up to a global $GL(2,\mathbb{C})$ transformation) the *same* perturbed vacuum spinor fields (20) that factor the Clifford tetrads must be used as *moving spin frames* for the (*spin*-$\frac{1}{2}$) *matter fields* [6], [14], [19], in order to preserve covariance of the Dirac equations in curved spacetime!

This is the principle of "general covariance", or

> *Spin Principle,* **S***: Spinor fields* are *physical*. All matter and gauge fields are (sums of) tensor products of spinor fields and their differentials [2], [9], [19], [20].

Spinors come in $2^3 = 8$ varieties: *left* or *right-chirality* (handed-nes, i.e. $su(2)$ phase rotation with spatial translation), *heavy* or *light temporality* (outward or inward propagation in cosmic time, T), and positive or negative *charge* ($u(1)$ phase shift with cosmic time).

Spinors are real [2]. Material particles are localized configurations of spinor fields; spacetime is the homgeneous distribution of the vacuum spinors between particles. Our moving spacetime tetrads $E_\alpha(x) \in T\mathbb{M}(x)$ *are* the inverse images under spinorization maps (15) of the Clifford tetrads $q_\alpha(x)$ and $\bar{q}_\alpha(x)$: standing-wave distributions (17) of internal $gl(2,\mathbb{C})$ phase stepped off by the vacuum spinors.

Spinorization maps S and $\bar{S}$ are implemented *physically* by the *spin* connections

$$\begin{aligned} \Omega_L(x) \equiv g^L\mathbf{d}g_L &= \tfrac{i}{2}\left[a_{\#}^{-1}\sigma_\alpha(x) + W_\alpha(x)\right]e^\alpha \\ &= \tfrac{i}{2}\left[a_{\#}^{-1}q_\alpha(x) + w_\alpha(x)\right]E^\alpha, \\ \Omega_R(x) \equiv g^R\mathbf{d}g_R &= \tfrac{i}{2}\left[a_{\#}^{-1}\overline{\sigma}_\alpha(x) + \overline{W}_\alpha(x)\right]e^\alpha \\ &= \tfrac{i}{2}\left[a_{\#}^{-1}q_\alpha(x) + \overline{w}_\alpha(x)\right]E^\alpha, \end{aligned} \tag{21}$$

the *left invariant Maurer-Cartan forms*, given first in the fiducial $\mathbb{M}_{\#}$ reference frame, and then in the dilated (but static) frame

$$\begin{aligned} E^\alpha &= \gamma e^\alpha; \\ q_\alpha &= \gamma^{-1}\sigma_\alpha. \end{aligned} \tag{22}$$

In an intrinsic reference frame co-expanding with the Friedmann flow, the temporal spin connections

$$\Omega_0^{\pm} = \frac{i}{2a_{\#}}\left(\pm i + i\dot{y}_0\right)$$

pick up negative "kinetic energy" terms in $\frac{\dot{y}^0}{2a_{\#}} = \frac{\dot{\gamma}}{2}$, the *rate* of cosmic expansion. This might be thought of as a sort of "Doppler shift" of energies in our expanding frame relative to the static reference frame (22) on $\mathbb{R}_+ \times \mathbb{S}_3\,(a)$.

4 The Topological Lagrangian and its Spinor Factorization

In the PT-symmetric (gravitational) case, the electroweak vector potentials w_α and $\overline{w}_\alpha$ in (21) vanish, and the spin connections *are* the tetrads. The exterior product of all 4 spin connections is a natural *topological Lagrangian*, whose action is the covering number of spin space over spacetime.

Products of cospinors and spinor differentials make *effective spin connections:* Lie-algebra valued 1 forms (3) that give the internal chiral $gl\,(2,\mathbb{C})$ phase increments stepped off over each infinitesimal spacetime displacement.

Now it takes tensor products of **4** pairs of cospinors and spinor differentials to make a *natural* 4 form—e.g. a Lagrangian density 4 form

$$\mathcal{L}_g \in \Lambda^4 \subset \otimes^8 \tag{23}$$

invariant under the group E_P of passive spin isometries in curved spacetime. In the PTC-symmetric case, the wedge product of all four of the Hermitian spin connections (21) makes the invariant 4-*volume* element—a Clifford scalar [2]:

$$\begin{aligned}\tfrac{i}{2}Tr\Omega^R \wedge \Omega_L \wedge \Omega^L \wedge \Omega_R &= \left(\tfrac{1}{16a_{\#}^4}\right)\sigma_0 e^0 \wedge e^1 \wedge e^2 \wedge e^3 \equiv \left(\tfrac{1}{16a_{\#}^4}\right)\sigma_0 \mathbf{d}^4 v \\ &= \left(\tfrac{1}{16a_{\#}^4}\right)|g|^{\frac{1}{2}}\,\sigma_0 E^0 \wedge E^1 \wedge E^2 \wedge E^3 \equiv \left(\tfrac{1}{16a_{\#}^4}\right)|g|^{\frac{1}{2}}\,\sigma_0 \mathbf{d}^4 V.\end{aligned} \tag{24}$$

Here $|g|^{\frac{1}{2}}$ is the square root of (minus) the determinant of the covariant metric tensor $g_{\alpha\beta}$ of (16): the inverse expansion factor of a 4-volume element on $\mathbb{M}$ comoving with Friedmann flow[2] relative to the unit volume element on $\mathbb{M}_{\#}$.

The *simplest* Lagrangian that is an E_P-invariant 4 form is thus the *topological Lagrangian*

$$\mathcal{L}_T \equiv \frac{i}{2}Tr\Omega^R \wedge \Omega_L \wedge \Omega^L \wedge \Omega_R; \tag{25}$$

[2]For a *static* frame $E^\alpha = \gamma e^\alpha$, $|g|^{\frac{1}{2}} = \gamma^{-4}$. For a Lorenz frame expanding at rate $\dot{y}^0 = a_{\#}\dot{\gamma}$, the comoving 1 forms are $E^{\alpha\prime} = \left(1 + a_{\#}^2\dot{\gamma}^2\right)^{\frac{1}{2}} E^\alpha$, and the inverse 4-volume expansion factor is

$$|g'|^{\frac{1}{2}} = \left(1 + a_{\#}^2\dot{\gamma}^2\right)^2 |g|^{\frac{1}{2}} \equiv \gamma'^{-4}\,|g|^{\frac{1}{2}}\,.$$

the *Maurer-Cartan* 4 form. The *topological action*

$$S_T \equiv \frac{i}{2}\int_{\widehat{\mathbb{M}}} Tr\Omega^R \wedge \Omega_L \wedge \Omega^L \wedge \Omega_R = -16\pi^3 C_2 \tag{26}$$

measures the *covering number* (second Chern number) of spin space over the *regular region* [3], [9]: the perturbed vacuum $\widehat{\mathbb{M}} \equiv \mathbb{M}_\# \backslash \cup D^J$ outside the codimension-J *singular loci;* the supports γ_{4-J} of massive particles.

More generally, our base spacetime $\mathbb{M}_\#$ is *stratified* into the *regular stratum* $D^0 = \widehat{\mathbb{M}} \equiv \mathbb{M}_\# \backslash \cup D^J$, where geometrical optics ansatz (1) holds for all 4 PTC-opposed pairs of spinor fields, and co-dimension-J *singular strata.* Here, the phases of $J = 1, 2, 3$, or 4 pairs of spinors either become singular, or break away from PTC-symmetry [3].

We call the union $\cup D^\alpha$ of all these strata and their incidence relations, together with the periods of all 1, 2, 3, and 4 forms quantized over embedded homology cycles, the *Spin*^{c}*-4 complex,* S.

Each singular locus $D^J \subset S$ carries its own topological charges: integrals of J forms or their dual $(4-J)$ forms over J cycles or their transverse $(4-J)$ cycles [3]. Each charge is quantized in integer units. The spin connections (7) derived from the canonical maps (6) of spin space over spacetime correspond to covering number $C_2 = 1$.

The topological action S_T is a *conformal invariant* that comes in topologically-quantized units [3]. Any (local) rescaling of a solution preserves S_T,and so is still a solution. Thus, topological Lagrangian $\mathcal{L}_T$ cannot be the Lagrangian for massive fields *inside* their worldtubes, because *mass* arises from the breaking of scale invariance.

Elsewhere [1], [2] we exhibited a "grandparent" Lagrangian density for both the *outer* (regular) region [3], where it reduces to $\mathcal{L}_T$, and the *inner* (singular) region, inside the worldtubes of massive particles:

$$\mathcal{L}_g \equiv i\mathbf{d}\psi^{R\pm}\psi_{L\mp} \wedge \psi^{L\pm}\mathbf{d}\psi_{R\mp} \wedge \mathbf{d}\psi^{L\pm}\psi_{R\mp} \wedge \psi^{R\pm}\mathbf{d}\psi_{L\mp} \tag{27}$$

(average over all neutral sign combinations in which each spinor, or its differential, appears exactly *once*). $\mathcal{L}_g$ is the **8***-spinor factorization* of the *Maurer-Cartan* 4 *form.*

One remarkable feature of $\mathcal{L}_g$ is [2], [16] that it yields effective electroweak, strong, and gravitational *field actions* in $\widehat{\mathbb{M}}$ when each field is expanded as the sum of a vacuum (dark energy) distribution with vacuum amplitudes of order $\gamma^{-\frac{1}{2}}$, and a "broken out" matter spinor:

$$\begin{array}{ll} \psi_{L\pm} \equiv \gamma^{-\frac{1}{2}}\widehat{\ell}_\pm + \xi_\pm, & \psi_{R\pm} \equiv \gamma^{-\frac{1}{2}}\widehat{r}_\pm + \eta_\pm, \\ \psi^{L\pm} \equiv \gamma^{-\frac{1}{2}}\widehat{\ell}^\pm + \chi^\pm, & \psi^{R\pm} \equiv \gamma^{-\frac{1}{2}}\widehat{r}^\pm + \zeta^\pm. \end{array} \tag{28}$$

All **8** fields in $\mathcal{L}_g$ may be varied independently. *Outside* the worldtubes D_J of massive particles, the action S_g is stationarized by the *outer solution.* This turns out [2], [6] to be both C and PT-symmetric: it preserves the "inner" products $\psi^{R\pm}\psi_{L\mp}$ of PTC-conjugate spinors as conformal invariants [2]. Under

PTC symmetry [2] in $\widehat{\mathbb{M}}$, outside the worldtubes of massive particles, we may use the simpler form

$$\mathcal{L}_g \overset{PTC}{\Longrightarrow} i\gamma^{-2}\mathbf{d}\psi^{R\pm} \wedge \mathbf{d}\psi_{R\mp} \wedge \mathbf{d}\psi^{L\pm} \wedge \mathbf{d}\psi_{L\mp} \tag{29}$$

for the field Lagrangian.

What happens *inside* the worldtubes B_4 of massive particles [2] is that Left- and Right-chirality of matter spinors, which have opposite nonAbelian magnetic charges, *bind* to form localized PT-antisymmetric *bispinor particles* like

$$e_- \equiv \left(\xi_-(x) \oplus \eta_-(x)\right), \tag{30}$$

the *electron.*

5 Vacuum Spinors and Dirac Mass

Mass arises from the breaking of scale invariance. The mechanism that breaks scale invariance and endows bispinor particles with *inertial mass* emerges in a remarkable way [2], [6] when ansatz (28) for the perturbed spinor fields is inserted into Lagrangian (27). We summarize below (please see Appendix also).

Massless fields, like the vacuum spinors in (28), contribute terms with conformal weight γ^{-4} to the Lagrangian. These integrate to terms of $O\left(\gamma^0\right)$—constant terms—like homogeneous the vacuum action of $-16\pi^3 W$ (26).

As the scale factor γ increases past 1, terms of $O\left(\gamma^{-3}\right)$ in $\mathcal{L}_g$ localize about 3 cycles, corresponding to the condensation of bispinor particles (leptons) from "seed" perturbations wrapped about 3 cycles in the perturbed vacuum. It is these $O\left(\gamma^{-3}\right)$ terms that give the effective *massive Dirac action* [2]. Heuristically, what happens is this.

Inside the worldtube B_4 of the massive bispinor particle e_-, the PTC-opposed pairs of matter fields $\left(\xi_-, \zeta^+\right)$ and $\left(\eta_-, \chi^+\right)$ of (28) undergo *mass scatterings* [9]: *Spin*^{c}-4 resonances with the remaining vacuum fields and differentials, which pair to form effective spin connections (9):

$$\begin{gathered} i\hat{\Omega} \wedge \hat{\Omega} \wedge \hat{\Omega} \wedge \zeta^+ \mathbf{d}\xi_- \\ i\left(\frac{i}{2a_\#}\right)^3 q_1\overline{q}_2\overline{q}_3 E^1 \wedge E^2 \wedge E^3 \wedge \zeta^+ D_0\xi_- E^0 \\ = \left(\frac{1}{2a_\#}\right)^3 \gamma^{-1}\sigma_1\gamma^{-1}\overline{\sigma}_2\gamma^{-1}\overline{\sigma}_3 E^1 \wedge E^2 \wedge E^3 \wedge \zeta^+ D_0\xi_- E^0 \\ = i\left(\frac{1}{2a_\#}\right)^3 \gamma^{-3}\sigma_0\zeta^+ D_0\xi_- \mathbf{d}^4 V. \end{gathered}$$

Here $D_\alpha\xi = \gamma^{-1}\partial_\alpha\xi$ are covariant derivatives (2) in the *dilated* coframe, $E^\alpha = \gamma e^\alpha$, or $E^{\alpha\prime} = \gamma' e^\alpha$ in a *coexpanding* coframe. Each *Spin*^{c}-4 resonance reconstructs a 3-*volume element* dual to the light spinor gradient, giving the intrinsic Dirac operators $i\overline{\sigma}_\alpha\nabla_\alpha$ and $i\sigma_\alpha\overline{\nabla}_\alpha$ [2].

Meanwhile, multilinear (tensor) products of 4 vacuum differentials and 2 vacuum spinors produce *spin*-1 tensors like

$$\begin{array}{c}\mathbf{d}\widehat{r}_{\pm}\widehat{\ell}^{\mp}\wedge\widehat{\ell}_{\pm}\mathbf{d}\widehat{r}^{\mp}\wedge\mathbf{d}\widehat{\ell}^{\mp}\otimes\mathbf{d}\widehat{\ell}_{\pm}\equiv-\left(\frac{i}{2a_{\#}}\right)^{4}\sigma_{0}\left(x\right)d^{4}V\\ \mathbf{d}\widehat{\ell}_{\pm}\widehat{r}^{\mp}\wedge\widehat{r}_{\pm}\mathbf{d}\widehat{\ell}^{\mp}\wedge\mathbf{d}\widehat{r}^{\mp}\otimes\mathbf{d}\widehat{r}_{\pm}\equiv\left(\frac{i}{2a_{\#}}\right)^{4}\overline{\sigma}_{0}\left(x\right)d^{4}V.\end{array} \tag{31}$$

These couple pairs of light and heavy matter spinors, contributing the effective *mass* term,

$$\mathcal{L}_{M}=-\left(\frac{i}{2a_{\#}}\right)^{4}\left[\zeta^{+}\sigma_{0}\left(x\right)\eta_{-}-\chi^{+}\overline{\sigma}_{0}\left(x\right)\xi_{-}\right]\gamma^{-3}\mathbf{d}^{4}V, \tag{32}$$

to complete the Dirac Lagrangian $\mathcal{L}_D$ [2].

Upon variation with respect to the heavy (light) spin vectors, $\mathcal{L}_D$ gives the *massive Dirac equations* coupling the light (heavy) envelopes of matter spinors [2], written with respect to a moving frame of unit spin matrices $\sigma^{\alpha}(x)$, $\overline{\sigma}^{\alpha}(x)$ and covariant derivatives $\nabla_{\alpha}(x)$, $\overline{\nabla}_{\alpha}(x)$ intrinsic to our dilated Friedmann 3-brane $S_3(T)$:

$$\begin{array}{ll}i\sigma^{\alpha}\boldsymbol{\nabla}_{\alpha}\boldsymbol{\xi}_{-}=\frac{\beta^{-1}}{2a_{\#}}\boldsymbol{\eta}_{-}, & i\left(\boldsymbol{\nabla}_{\alpha}\boldsymbol{\chi}^{+}\right)\sigma^{\alpha}=\frac{\beta}{2a_{\#}}\boldsymbol{\zeta}^{+}\\ i\overline{\sigma}^{\alpha}\overline{\boldsymbol{\nabla}}_{\alpha}\boldsymbol{\eta}_{-}=\frac{\beta^{-1}}{2a_{\#}}\boldsymbol{\xi}_{-}, & i\left(\overline{\boldsymbol{\nabla}}_{\alpha}\boldsymbol{\zeta}^{+}\right)\overline{\sigma}^{\alpha}=\frac{\beta}{2a_{\#}}\boldsymbol{\chi}^{+}.\end{array} \tag{33}$$

These govern the lightest perturbations that unfold from the conformal vacuum as γ exceeds 1 (see Appendix). The electron mass [6],

$$m_{e}=\frac{\beta}{2a_{\#}}, \tag{34}$$

turns out to be the inverse of the equilibrium diameter $2a_{\#}$ of our Friedmann 3-brane [6], [15] in the $\mathbb{M}_{\#}$ ($\beta=1$) reference frame. In an intrinsic frame coexpanding with the Friedmann flow $\dot{y}^{0}\equiv\dot{T}$,

$$\beta=\left[\frac{1+\dot{T}}{1-\dot{T}}\right].$$

The mass (34) of a “free” electron is an example of *quantization of action:* a 1 form integrated over a 1-cycle $\gamma_1\in\mathbb{M}_{\#}$ *transverse* to the electron’s 3-support $*D^{1}=B_{3}$:

$$\oint_{\gamma_{1}}Edt-\mathbf{p}d\mathbf{x}=\int_{0}^{2\pi a_{\#}}m_{e}dt=\pi$$

(where $\hbar$= 1).

The worldtube $B_4=B_3\times\gamma_1$ of a massive Dirac particle shares a common boundary $\partial B_4\in\partial\widehat{\mathbb{M}}$ with the perturbed vacuum $\widehat{\mathbb{M}}$. On a microscopic scale,

(Section 8) it is *mass scatterings* on ∂B_4—the *discrete* form of *Spin*^{c}-4 resonances (32) with the vacuum spinor fields (31)—that channel the "null zig-zags" of the Dirac propagator [8], [9], [16] into a timelike worldtube B_4. Inside B_4, the light spinors $(\xi_-,\ \eta_-)$ zig zag forward (outward) in cosmic time T, while the heavy spinors $(\chi^+,\ \zeta^+)$ zig zag backward (inward), producing a net drift that is only *statistically* forward in T. At each vertex where a light and heavy spinor meet, the *photons* $\gamma^{\circlearrowleft} \equiv \xi_- \otimes \zeta^+$ or $\gamma^{\circlearrowright} \equiv \eta_- \otimes \chi^+$ appear to be created or annihilated. It is the statistics of this process—the stochastic interactions of four matter spinors with four vacuum spinors—that yields the massive Dirac-Maxwell propagator of Q.E.D. *exactly* [8].

Meanwhile, it is easy to see why no massive particle can move faster than light—which is "all zig, and no zag": because its internal zigs and zags, though all lightlike, propagate in opposite directions!

The *vacuum fields* create *Dirac mass*—the resistance of a bispinor particle to acceleration. Qualitatively, when a particle, P, is accelerated by $\triangle v$ over one mass scattering time $\triangle t$, the vacuum spinors impart a greater momentum change $\triangle p \equiv p^- - p^+$ to the "trailing surface" ∂B_4^- of its worldtube boundary than to its "leading surface" ∂B_4^+. The *frequency* of mass scatterings is (proportional to) the rest mass of P; $(\triangle t)^{-1} \sim m$. Thus $\triangle p \sim m \triangle v$; the discrete version of Newtons law for inertial forces. This is Mach's principle in action. Quantitatively, all of the relativistic kinematics of massive particles may be derived from mass scatterings with the vacuum spinor fields on the worldtube boundary [24].

Reciprocally, massive particles perturb the vacuum fields. On the macroscopic level, *gravitation* is the spacetime curvature on the boundary of the world tube of a test particle caused by perturbations due to sources. We derive Einstein's field equations in the next section by matching the *spin curvatures* due to the source to the *stress-energy* on the moving boundary ∂B_4 of a test particle's worldtube.

6 Matching Boundary Vorticity to Energy Momentum Flux

The form of the effective matter Lagrangian 4 form, $\mathcal{L}_M$, *inside* the worldtube B_4, and its stress-energy 3 form $*T$, depends on the particle. But the form of the *field* Lagrangian outside the worldtubes is universal. This gives us just enough information to match the integral of the *outer, field* 3 form G, and the *inner, energy-momentum* flux $*T$ of the matter fields on the *moving boundary* $\partial B_4\,(\tau)$, and thus *derive* Einstein's field equations. We outline the steps below. For convenience, we work in our fiducial reference frame $e^\alpha \in T^*\mathbb{M}_\#$; then translate our results to a dilated frame $E^\alpha = \gamma e^\alpha \in T^*\mathbb{M}$.

1. Write the total action S_g as the sum of the *field* terms *outside* $B_4\,(\tau)$ (in

$\widehat{\mathbb{M}} \equiv \mathbb{M}_{\#} \backslash B_4$), and *matter* terms *inside*:

$$S_g = \frac{i}{2} \int_{\widehat{\mathbb{M}}} Tr \Omega^R \wedge \Omega_L \wedge \Omega^L \wedge \Omega_R + \int_{B_4(\tau)} \mathcal{L}_M \equiv S_F + S_M. \qquad (35)$$

Here τ is a proper time parameter along the particles' world tubes, projected to $\mathbb{M}_{\#}$ [6].

2. Transform the *field* term via integration by parts using the Bianchi identity

$$\mathbf{d}K = K \wedge \Omega - \Omega \wedge K.$$

The result is [2]

$$\begin{array}{c} S_F = \frac{i}{2} \int_{\widehat{\mathbb{M}}} Tr \left[K_L \wedge K_R + G_L \wedge G_R \right] - Tr \left[\Omega_L \wedge \left(K_L + K_R \right) \wedge \Omega_R + P \right] \\ -i \int_{\partial B_4(\tau)} Tr \left[\Omega_L \wedge K_R + K_L \wedge \Omega_R \right], \end{array} \qquad (36)$$

where P means space reversal. The first term is the chiral version of the action in the electroweak and strong fields [2], [16]. These may be combined to make the net (left) spin curvature 2 form

$$H_L \equiv K_L \oplus G_L \equiv \left(K^0_{L\gamma\delta} \sigma_0 \oplus K^j_{L\gamma\delta} \sigma_j \oplus G^{jk}_L \sigma_j \otimes \sigma_k \right) e^\gamma \wedge e^\delta \qquad (37)$$

(and similarly for H_R, with the σ_α replaced with $\overline{\sigma}_\alpha$). H_L and H_R take their values in the *tensor product* $gl\left(2, \mathbb{C}\right)_\uparrow \otimes gl\left(2, \mathbb{C}\right)_\downarrow$ of *heavy* (baryonic) and *light* (leptonic) Lie algebras. This decomposes into the direct sum

$$gl\left(2\right)_\uparrow \otimes gl\left(2\right)_\downarrow = \mathbb{C} \left[u\left(1\right) \oplus \left(su\left(2\right)_\uparrow \oplus su\left(2\right)_\downarrow \right) \oplus su\left(3\right) \right]$$

of complexified $u\left(1\right)$ (electromagnetic), $su\left(2\right)$ (weak; baryonic $\oplus$ leptonic), and $su\left(3\right)$ (strong) Lie algebras [2].

The *electroweak* fields come from the *antiHermitian* parts $\frac{i}{2} d\theta^\alpha \sigma_\alpha$ of the $gl\left(2, \mathbb{C}\right)$-valued vector potentials [9]. Their *Hermitian* parts $\frac{1}{2} d\varphi^\alpha \sigma_\alpha$ give the second term in (36), which contains the *Palatini action* for the *gravitational field* $K \equiv K_L \oplus K_R$. *Both* the Hermitian and nonHermitian potentials contribute to strong (G) fields in (37). But it is the third term—the *boundary integral* term in field action (36)—that couples fields to source currents in the next steps.

3. Express the boundary integral in terms of the matrix-valued *spacetime curvature* 2 form [13]

$$\mathcal{R}_\alpha^{\ \beta} \equiv R_{\alpha\ \gamma\delta}^{\ \beta} e^\gamma \wedge e^\delta. \qquad (38)$$

$\mathcal{R}$ accepts the area element $\left[e_\gamma, e_\delta\right]$ and returns the holonomy *(rotation)* matrix around it, with matrix elements $\mathcal{R}_\alpha^{\ \beta}$.

4. Rewrite all spacetime vectors as Clifford (C) vectors, using spinorization maps (15), with $q_\alpha = \sigma_\alpha$ on $T\mathbb{M}_\#$. Now re-express the PT-symmetric part of the spacetime curvature matrix, acting on a basis C vector, in terms of C vectors multiplying the $gl\,(2,\mathbb{C})$-valued *spin-curvature* 2 forms [14], [16]:
$$\mathcal{R}_\alpha^{\ \beta}\sigma_\beta = \sigma_\alpha K_R + K_L\overline{\sigma}_\alpha, \tag{39}$$
where
$$\begin{aligned} K_L &\equiv K_{L\ \beta\gamma}^{\ \alpha}\sigma_\alpha e^\beta \wedge e^\gamma, \\ K_R &\equiv K_{R\ \beta\gamma}^{\ \alpha}\overline{\sigma}_\alpha e^\beta \wedge e^\gamma. \end{aligned} \tag{40}$$

5. Using Cartan's C vector-valued 1 form,
$$\mathbf{d}q\,(x) \equiv \mathbf{d}\,(\sigma_\alpha x^\alpha) \equiv \sigma_\alpha e^\alpha, \tag{41}$$
recognize the trace of the $gl\,(2,\mathbb{C})$-valued 3 form
$$\begin{aligned} \left(\tfrac{1}{2a_\#}\right) G &\equiv \tfrac{1}{2a_\#}\mathbf{d}q \wedge \mathcal{R} = -i\,[\Omega_L \wedge K_R + K_L \wedge \Omega_R] \\ &\equiv \left(\tfrac{1}{2a_\#}\right) G^\alpha_{\ \beta}\sigma_\alpha \epsilon^\beta_{\ \gamma\delta\mu} e^\gamma \wedge e^\delta \wedge e^\mu \end{aligned} \tag{42}$$
as the *integrand* in the *outer form* of the boundary integral in (36). The C-vector-valued 3 form G is Wheeler's [13] "moment of rotation tensor": the "vorticity" $\mathcal{R}$ of the *spinfluid dilation-boost flow* times the normal *moment arm* to the area element.

6. The *inner* form of the boundary integral is the energy-momentum C vector $\mathcal{P} \equiv \mathcal{P}^\alpha\sigma_\alpha$ of the matter fields inside the worldtube B_4 of the moving particle. Detect this by displacing B_4 by the spacetime increment $t \equiv \triangle x^\alpha$ and rewriting the change in the action as the *surface* integral of a C-vector-valued 3 form *flux,* $*T \equiv *T^\alpha\sigma_\alpha$, across the moving boundary [11], [12]:
$$\mathcal{P}^\alpha\,(t) \equiv \int_{\partial B_4(t)} *T^\alpha. \tag{43}$$
Here
$$*T^\alpha \equiv \left[\left(\frac{\partial\mathcal{L}}{\partial\,(\partial_\alpha\psi_I)}\right)\partial_\beta\psi_I - \delta^\alpha_\beta\mathcal{L}\right] * e^\beta \equiv T^\alpha_\beta * e^\beta, \tag{44}$$
where
$$*e^\beta \equiv \epsilon^\beta_{\ \gamma\delta\mu} e^\gamma \wedge e^\delta \wedge e^\mu$$
is the 3 form Hodge dual to the unit 1 form e^β.
$$*T \equiv T^\alpha_\beta\sigma_\alpha * e^\beta \in \Lambda^3\,(\mathbb{M}_\#)$$
is a C-vector valued 3 form: the *energy-momentum* 3 form. $*T^\alpha$ is the Noether current under translation in the e_α direction. Taking $t = \tau$, the proper time along a particles worldline, (43) gives $\mathcal{P}^0\,(\tau)$: the *energy* contained in the particle's spatial support $B_3\,(\tau)$; a Clifford *scalar.* This

is its *rest mass.*

More generally, if $\mathcal{L}$ is t-translation invariant, the action contained *inside* the closed worldtube $B_4 \equiv \mathbb{S}_1(t) \times B_3 \subset \mathbb{M}_{\#}$ on compactified Minkowsky space is

$$\mathcal{S}_M(B_4) = \int_{\mathbb{S}_1} dt \int_{B_3(t)} *T = 2\pi a_{\#} \int_{B_3} *T. \tag{45}$$

7. Finally, equate the inner and outer expressions (44) and (42) in the action integral over the moving boundary

$$\partial B_4(t) \equiv B_3(t) - B_3(0) + S_2 \times I(t)$$

of a section of the worldtube of a particle in an external field. In the particles rest frame, which is "freely falling" in the external field, the momentum flux over the boundary $S_2 \times I(\tau)$ vanishes, and we obtain:

$$\begin{array}{c} \frac{1}{2a_{\#}} \int_{B_3(\tau)} G = 2\pi a_{\#} \int_{B_3(\tau)} *T \Longrightarrow G = 4\pi a_{\#}^2 * T; \\ \text{i.e. } G^{\alpha}_{\ \beta} = 4\pi a_{\#}^2 T^{\alpha}_{\ \beta} \end{array} \tag{46}$$

componentwise, since both integrals must be E_P-invariant. These are *Einstein's field equations* [13] on $\mathbb{M}_{\#}$, with a gravitational constant of

$$\kappa = \frac{a_{\#}^2}{2}, \tag{47}$$

the mean squared radius of the equilibrium Friedmann solution.

Finally, we translate equation (46) to the dilated spacetime frame $E^{\alpha} = \gamma e^{\alpha} \in T^*\mathbb{M}$ and Clifford-algebra frame $q_{\alpha} = \gamma^{-1}\sigma_{\alpha} \in C(T\mathbb{M})$. Note (39), (40), (41), (42) that G is a "Clifford Trivector" [13]; it contains products of 3 C vectors $q_{\alpha}q_{\beta}q_{\gamma} = \gamma^{-3}\sigma_{\alpha}\sigma_{\beta}\sigma_{\gamma}$ (since the spin curvature 2 forms contain products of 2 C vectors). Thus,

$$G^{\alpha\prime}_{\ \beta} = \gamma^{-3} G^{\alpha}_{\ \beta}$$

on $\mathbb{M}$. Since $*T \equiv T^{\alpha}_{\ \beta}\sigma_{\alpha} = T^{\alpha\prime}_{\ \beta}\gamma q_{\alpha}$ is an ordinary C vector,

$$T^{\alpha\prime}_{\ \beta} = \gamma^{-1} T^{\alpha}_{\ \beta}$$

on $\mathbb{M}$. Thus,

$$G^{\alpha\prime}_{\ \beta} = \gamma^{-2} 4\pi a_{\#}^2 T^{\alpha\prime}_{\ \beta}; \tag{48}$$

Einstein's field equations on $\mathbb{M}$, with a gravitational constant of

$$\kappa' = \gamma^{-2} \frac{a_{\#}^2}{2} \tag{49}$$

in a dilated, but static-frame. In a frame comoving with cosmic expansion, γ is replaced by

$$\gamma' \equiv \left(1 + a_{\#}^2 \dot{\gamma}^2\right)^{\frac{1}{2}} \gamma.$$

Note that the gravitational constant *decreases* with radius $a(t) = \gamma(t)\, a_{\#}$ of our Friedmann 3-surface, and with the Hubble constant $\frac{\dot{a}}{a}(t) = \frac{\dot{\gamma}}{\gamma}(t)$. Perhaps such effects could be detected in astronomical data.

An independent check on our value (47) for κ on $\mathbb{M}_{\#}$ is provided by balancing $*T$ on the inside versus G on the outside of $B_3 \equiv \mathbb{S}_3(a_{\#})$, the *equilibrium* Friedmann 3-sphere. Here the curvature 2 form and Cartan unit 1 form are

$$\mathcal{R}^i{}_j = \delta^i{}_\ell \delta_{jp} e^\ell \wedge e^p; \quad \mathbf{d}q \equiv \mathbf{d}\left(a_{\#} \exp \frac{i}{2a_{\#}} x^j \sigma_j\right).$$

This gives the moment-of-rotation 3 form (42)

$$G = a_{\#} \epsilon^{\;n}_{jk\ell} \exp\left(\frac{i}{2a_{\#}} x^m \sigma_m\right) \sigma_n e^j \wedge e^k \wedge e^\ell;$$

the area 3 form on $\mathbb{S}_3(a_{\#})$ times the *normal* C vector.

Meanwhile, the C-vector-valued 3 form $*T$ must integrate to a constant—the rest energy in $\mathbb{S}_3(a_{\#})$. $*T$ must be proportional to $a_{\#}^{-3}$ and, like G, be Clifford *normal* to $\mathbb{S}_3(a_{\#})$. We thus obtain relations (46) and (47) directly, by balancing the outward pressure $*T$ of cosmic expansion against the inward restoring force due to the extrinsic curvature G of our embedded Friedmann 3-brane $\mathbb{S}_3(a_{\#}) \subset \mathbb{R}_4$.

Much as the radius of a soap bubble reflects a balance between internal pressure and extrinsic curvature, the value $\kappa = \gamma^{-2} \frac{1}{2} a_{\#}^2$ of the gravitational constant is a local "memory" of the global balance between pressure and curvature that sets the equilibrium radius of our Friedmann universe.

Gravity is attractive because neighboring dilation currents $d\varphi^0$ (masses) present *opposite* radiotemporal vorticities

$$K^0 \equiv \left(K^0_R + K^0_L\right) = \frac{1}{2}\left[\partial_0, \partial_r\right]\left(\varphi^0_L + \varphi^0_R\right) e^0 \wedge e^r \tag{50}$$

to each others' worldtube boundaries. To minimize the net vortex energy, they *advect*, like counter-rotating hydrodynamic vortices [11]; i.e. attract, orbit around each other—and perhaps fuse.

When massive particles (e.g. protons and neutrons) which contain both leptonic (light) and baryonic (heavy) spinors get very close together ($r \longrightarrow 0$), the interaction term $TrG_L \wedge G_R \sim r^{-8}$ in the *strong* fields in (37) begins to dominate the action (36). Just as electromagnetic and weak curvatures unify to make up the *antiHermitian* (PT-antisymmetric, or charge-separated) parts K_A of the net spin curvature 2 form (37), *gravitational* and *strong* curvatures unify to make the *Hermitian* (PT-symmetric, or neutral) part $K_H \oplus G$. We call $(K_H \oplus G)$ the *gravitostrong* field.

7 Gravitomagnetic-Nuclear Couplings and Axial Jets

The field ($\widehat{\mathbb{M}}$) integral in (36) is the action of the *perturbed vacuum spinors* outside the worldtubes of massive particles. It gives the standard effective kinetic energy terms for electroweak and gravitostrong fields [2]. Standard coupling constants like κ' (49) are thus determined by the vacuum-field amplitudes, and hence their values depend on the radius of the Friedmann solution.

We show below how the field term in action (36) contains not only the standard *Palatini action* for the gravitational field, but predicts *a new interaction between gravitomagnetic fields and weak potentials*. This belongs neither to the electroweak nor gravitostrong sectors, but lives in the overlap domain demanded by their unification.

The new interaction terms arise by expanding the $\Omega K \Omega$ term in (36), using identities (21). Defining $K \equiv K_L + K_R \equiv K_{\alpha\beta} e^\alpha \wedge e^\beta$ as the *Hermitian* (*PT*-symmetric) part of the $gl\,(2,\mathbb{C})$-valued spin curvature 2 form, we obtain

$$\begin{aligned} &\tfrac{i}{2} Tr\left[\Omega_L \wedge K \wedge \Omega_R + P\right] \\ = &\tfrac{i}{2} Tr\left[\tfrac{1}{4} a_\#^{-2} q_\alpha K^{\alpha\beta} \overline{q}_\beta + \tfrac{1}{2} a_\#^{-1} q_\alpha K^{\alpha\beta} \overline{W}_\beta + W_\alpha K^{\alpha\beta} \overline{W}_\beta + P\right] \mathbf{d}^4 v \end{aligned} \tag{51}$$

in $\mathbb{M}_\#$ coordinates: The components $K^{\alpha\beta}$ of the *dual* spin curvature,

$$*K = \frac{1}{2} K_{\alpha\beta} \epsilon^{\alpha\beta} \gamma\delta \; e^\gamma \wedge e^\delta, \tag{52}$$

appear because it takes all four 1 forms and all four Clifford tetrads to make the invariant volume element, a Clifford *scalar:*

$$\sigma_0 e^0 \wedge \sigma_1 e^1 \wedge \sigma_2 e^2 \wedge \sigma_3 e^3 = i \sigma_0 \mathbf{d}^4 v = i \left|g\right|^{\frac{1}{2}} \sigma_0 \mathbf{d}^4 V.$$

Clifford multiplication is thus *dual* to exterior multiplication via the scalar product defined by the integral of the wedge product of Clifford-algebra-valued forms [3].

The term in $a_\#^{-2}\left[qK\overline{q} + P\right]\left|g\right|^{\frac{1}{2}} \mathbf{d}^4 V$ gives the *Palatini action* for gravitation [14]. Its variation with respect to the tetrads gives the Einstein field equations [14], with a gravitational constant of $\kappa' = \gamma^{-2}\frac{1}{2}a_\#^2$. The fact that we get the *same* value as (49) above provides an independent check on our boundary-integral method.

The terms in $a_\#^{-1}\left[qK\overline{W} + P\right]$ predict a new interaction between Lens-Thirring gravitational fields and weak potentials [21]. This is not a "fifth force", but an overlap between the standard interactions that lives in the overarching domain of their unification [2]. These new cross terms predict new physical phenomena—*axial jets* [21].

Suppose some weakly-decaying particles are located near the 3-axis of a disk, D, rotating about the 3 axis with angular velocity ω. The graviatational field

$$\begin{aligned} K &= K_{03} e^0 \wedge e^3 + K_{12}\left(\omega\right) e^1 \wedge e^2; \\ *K &= K_{03} e^1 \wedge e^2 - K_{12}\left(\omega\right) e^0 \wedge e^3 \end{aligned} \tag{53}$$

then contains the *gravitomagnetic* (Lens-Thirring) component $K^3_{12}(\omega)\sigma_3 = K^{03}_3\sigma_3$, whose magnitude depends upon ω. The new qKW cross term

$$\begin{aligned}&\tfrac{i}{2}Tr\Omega^0_0\sigma_0 e^0 \wedge K^3_{12}(\omega)\sigma_3 e^1 \wedge e^2 \wedge \tfrac{i}{2}\partial_3\left[\zeta^3_L - \zeta^3_R\right]\sigma_3 e^3 \\ &= \tfrac{1}{4}\Omega_0 K^{03}(\omega)\partial_3\left[\zeta^3_L - \zeta^3_R\right]\sigma_0 d^4v \equiv -V(\boldsymbol{\omega}\cdot\mathbf{W})\sigma_0 d^4v\end{aligned} \tag{54}$$

gives an effective potential in the dot product of the angular momentum vector $\boldsymbol{\omega}e_3$ and the nonAbelian vector potential $\mathbf{W}$, multiplying the *timelike part of the vacuum effective spin connection.* On a 3-brane $S_3(t)$, with (local) expansion rate [2], [6], [16]

$$\frac{\dot{a}}{a} = \frac{\dot{y}^0}{a_\#}, \tag{55}$$

this reads[3]

$$\widehat{\Omega}^{\pm}_0 e^0 = \frac{1}{2a_\#}\left(\pm i - \dot{y}^0\right)\sigma_0 e^0. \tag{56}$$

Letting the nonAbelian vector potential have both antiHermitian (imaginary) and Hermitian (real) parts,

$$\begin{aligned}W_L &= \tfrac{i}{2}\partial_3\zeta^3_L\sigma_3 e^3; \\ W_R &= \tfrac{i}{2}\partial_3\zeta^3_R\overline{\sigma}_3 e^3\end{aligned}$$

where $\zeta^3(x) \equiv \theta^3(x) + i\varphi^3(x)$, we obtain the real cross term

$$V(\boldsymbol{\omega}\cdot\mathbf{W}) = -\frac{1}{2a_\#}K^{03}(\omega)\left[\dot{y}^0\partial_3\left(\varphi^3_L - \varphi^3_R\right) \mp \partial_3\left(\theta^3_L - \theta^3_R\right)\right] \tag{57}$$

in $\mathcal{L}_g$.

In an expanding universe ($\dot{y}^0 > 0$), V decreases when the left-chirality parts of the weak-decay products are boosted more along the $\widehat{\boldsymbol{\omega}}$ direction than the right, independently of their charges; the second term describes a charge-dependent spin polarization. The net effect of the qKW interaction term is to cause *weak-decay products to be ejected with left-helicity with respect to the axis of rotation,* producing *axial jets.*

Astronomical observations often show plasma jets ejected along the axes of rotating quasars, pulsars, and active galactic nuclei [22]. I don't know if their helicity has been measured. Left-helical polarization of such jets would tend to support the NM model.

Now it could be argued that, if observed, the left-helical polarization could be explained by standard electroweak theory. But left-helicity enters there as an *assumption.* The NM model *derives* the left-helicity preference of weak decays from dynamical symmetry breaking of the vacuum spin connection Ω_0 in the *forward* timelike direction: $\dot{y}^0 > 0$ in (57). If our expanding Friedmann 3-brane

[3]Two pieces of evidence converge on a current (local) value of $\dot{y}^0 = 0.16$ (where $c = 1$). This predicts values of $\theta_W = 28.5°$ *and* of $\alpha = \frac{1}{137.6}$ for the Weinberg angle [16] and the fine structure constant [3], which match the observed values quite closely.

$S_3(t)$ is to remain bulk neutral (PT-symmetric), then when T is broken, P must be broken. The qKW term gives a *dynamical* mechanism for P-symmetry breaking on the cosmological scale.

On microscopic scales, the same qKW mechanism could drive the left-helicity weak-decay modes of massive spinning particles (e.g. nucleii). Even when there is no net spin, interaction with the (predominantly left helicity) vacuum spinors could drive left-helicity weak decays through the Newtonian field

$$K_{0r} = (*K)^{\theta\varphi} \equiv K^{\theta\varphi}$$

of a massive particle [4], via terms like $q_\theta K^{\theta\varphi} W_\varphi$.

$WK\overline{W}$ terms in (51) like

$$\begin{aligned} TrW_0 K^{03}\overline{W}_3 + P &= Tr\left(W_0 \otimes_A \overline{W}_3\right)\left(K^{03} \otimes \mathbf{1}\right) \\ &= Tr\left(W_0\overline{W}_3 - W_3\overline{W_0}\right)\gamma_{03}K^{03} \equiv TrG_{03}K^{03}, \\ \text{where} \quad \gamma_{03} &\equiv \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \equiv \gamma_3 - \gamma_8, \end{aligned}$$

seem to predict new interactions between gravitational and *strong* ($su(3)$-valued) fields: *strong* (nuclear fusion) jets emitted along the axis of a supermassive rotating "nucleus", like a neutron star. Meanwhile, the Newtonian gravitational field $*K_{0r} = K^{\theta\varphi}$ and antisymmetric tensor products of $su(2)$ potentials like $W_1 + iW_2 \equiv W_\theta^+$ and $\overline{W}_1 - i\overline{W}_2 \equiv W_\varphi^-$ combine to make new cross terms like

$$\begin{aligned} TrW_\theta K^{\theta\varphi}\overline{W}_\varphi + P &= TrG_{\theta\varphi}K^{\theta\varphi}; \\ G_{\theta\varphi} &\equiv \left[W_\theta\overline{W}_\varphi + W_\varphi\overline{W}_\theta\right]\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

These seem to predict accelerated nuclear reaction rates in a strong Newtonian field. I don't know if such effects are observed, or, if so, how standard models explain them.

The advantage of the *Spin*^{c}-4 model here is that it remains nonsingular in collapsed matter—the regime of *grand unification* (see Appendix)—where standard models break down.

8 The Singular *Spin*^{c}-4 Complex and Quantum Gravity

Let's return to our central point:

> *Nonlinear interaction of localized matter spinors through globallly-nontrivial vacuum spinor fields creates the inertial masses of the particles. The resulting perturbation to the vacuum spinors produces the gravitational interaction between particles.*

Let's see how this is implemented on the microscopic scale. Here, the mass scatterings that define the fuzzy boundary ∂B_4 of the particle's world tube are displaced by nonunitary perturbations of the vacuum spinors sourced in other particles. This is *quantum gravity.*

Quantum field theory is statistical mechanics in imaginary time [23], $T \equiv y^0$. T *is* cosmic time, which enters kinematically as the *imaginary* part of a complex time parameter $z^0 \equiv x^0 + iy^0$, whose real part $dx^0 \equiv |d\mathbf{x}|$ is arclength increment along a null ray (e.g. the path of a photon). The *energy* of a state (multiplied by the temperature) is replaced by the *action* of a path (divided by $i\hbar$) in all ensemble averages.

Note [2] that the 4 spinor fields $\left(\psi_{L-}, \psi_{R-}, \psi^{L+}, \psi^{R+}\right)$ are *analytic:* they obey the Cauchy-Riemann (CR) equations (10). The remaining 4 spinor fields $\left(\psi_{L+}, \psi_{R+}, \psi^{L-}, \psi^{R-}\right)$ are *conjugate analytic. It is these analyticity conditions that justify Wick rotation,* which translates the statistical mechanics of null zig-zags in *Euclidean spacetime,* with coordinates $\left(y^0, \mathbf{x}\right)$, to Feynman integrals over compactified Minkowsky space $\mathbb{M}_{\#}$, with coordinates $\left(x^0, \mathbf{x}\right)$ [23].

A stochastic version of our model, in which the classical action is replaced by a statistical propagator (the sum over null zig-zags [8], [9]) is thus a theory of quantum gravity, *provided* that the *vacuum spinors* that do the mass scatterings that confine the null zig-zags of massive particles to timelike worldtubes [9], [16], are also modelled statistically.

The chiral spinor fields that bind to form a massive particle are *lightlike:* each has *definite helicity* [9]. Their phases $\zeta^\alpha(z)$ or $\zeta^\alpha(\bar{z})$ may propagate *only* along segments of *forward* characteristics, $\gamma_+ : dy^0 = |d\mathbf{x}|$, or backward characteristics, $\gamma_- : dy^0 = -|d\mathbf{x}|$. To make a massive particle with *definite spin,* L- and R-chirality moieties must be *counterpropagating,* i.e. have opposite momenta, but the same spins, and therefore *opposite helicities.* The propagator for a massive bispinor particle [8], [9] is thus a *sum over null zig-zags:* counterpropagating lightlike segments with *mass scatterings* at each corner.

These mass scatterings are vertices in a Riemann sum for the action, S_g, which we compute as follows:

i) Re-express S_g with respect to complex Clifford (internal) and spacetime (external) *null tetrads* on $\mathbb{CM}_{\#} \subset T^*\mathbb{M}_{\#}$,

$$\begin{array}{ll} \sigma_\pm \equiv \frac{1}{\sqrt{2}}\left(\sigma_1 \pm i\sigma_2\right); & \sigma_{\uparrow\downarrow} \equiv \frac{1}{\sqrt{2}}\left(\sigma_0 \pm \sigma_3\right), \\ e^\pm \equiv \frac{1}{\sqrt{2}}\left(e^1 \pm ie^2\right); & e^{\uparrow\downarrow} \equiv \frac{1}{\sqrt{2}}\left(e^0 \pm e^3\right). \end{array} \tag{58}$$

ii) Discretize the **8**-spinor form (27) for our Lagrangian density $\mathcal{L}_g$, and compute our Riemann sums for S_g over *null lattice,* N, stepped off by null tetrads (58). The line segments of N are the (lightlike) rays of spinor fields. Its points (vertices) are *scattering events,* where m incoming chiral pairs of spinors—some of which may be vacuum spinors—scatter into $(4-m)$ outgoing pairs. These *Spin*^{c}*-4 scatterings* are the discrete versions of *Spin*^{c}-4 resonances like (31). The spinor differentials $\mathbf{d}\psi_I$ in $\mathcal{L}_g$

(27) are replaced by *first differences*

$$\psi_I(p+\triangle)-\psi_I(p)\equiv\triangle\psi$$

between neighboring lattice points in $\sum$. The "conjugate gradient" terms (9) are approximated by discrete $gl(2,\mathbb{C})$ phase differences:

$$\psi^I\mathbf{d}\psi_I=\frac{i}{2}\mathbf{d}\zeta_I^\alpha\sigma_\alpha\equiv\frac{i}{2}\mathbf{d}\zeta_I\sim\frac{i}{2}\triangle\zeta_I\equiv\frac{i}{2}\left(\triangle\theta^\alpha+i\triangle\varphi^\alpha\right)\sigma_\alpha. \tag{59}$$

Both the scalar ($\mathbb{C}u(1)$) parts

$$\triangle\zeta^0\equiv\left(\triangle\theta^0+i\triangle\varphi^0\right)\equiv\left(\triangle\text{charge}+i\triangle\text{energy}\right)$$

and the vector ($\mathbb{C}su(2)$) part

$$\triangle\zeta^j\equiv\left(\triangle\theta^j+i\triangle\varphi^j\right)\equiv\left(\triangle\text{spin}+i\triangle\text{momentum}\right)$$

contribute to the net $gl(2,\mathbb{C})$ phase increment $\triangle\zeta^\alpha(p)\,\sigma_\alpha$ at vertex p. However, to obtain an action that is a *real scalar* under all passive spin (E_P) transformations, we restrict our sums to the *scalar* (σ_0) component $\triangle\mathcal{L}_g^0\sigma_0$ of each *Spin*^{c}-4 scattering between J chiral pairs of matter spinors and $(4-J)$ vacuum pairs. The charges $\triangle\theta^0(p)$, spins $\triangle\theta^j(p)$, and 3 momenta $\triangle\varphi^j$ add to zero at every *Spin*^{c}-4 scattering, p.

iii) To automatically insure quantization of action [3], we should take the area of the elementary 2 cells $\widehat{\gamma}_2$ in a rectangular null lattice to be $\triangle p\triangle q=\frac{\hbar}{2}$. This not only assures that $\triangle\zeta(\gamma_2)=2\pi in$ about *any* phase space cycle γ_2, but also allows all of the spinor wave functions $\psi(p)$ to be *single-valued* at each $p\in N$. The singular homology $H_*(N)$ should be a skeleton for the homology $H_*\left(\widehat{\mathbb{M}}\right)$ of our spacetime manifold minus the singular loci [3]. Then all of the topological charges—J forms $D^J\in H^J\left(\widehat{\mathbb{M}}\right)$ quantized over cycles in $\widehat{\mathbb{M}}$—will have discrete realizations as products of J net phase differences quantized over discrete cycles in N. We call this complex of spinor phase shifts quantized over cycles in the null lattice N a *singular Spin*^{c}*-4* complex, $\sum$.

The action of each $\sum$ is the sum of contributions from every *Spin*^{c}-4 scattering in $\sum$. *Particle propagators* are sums over all null zig-zag paths in $\sum$ that connect the initial and final state [16].

Now the bending of the worldtube $B_4(P)$ of a massive particle P is described by a changing 4 momentum $\triangle\varphi^\alpha(P)=-\triangle\varphi^\alpha(V)$ imparted by mass scatterings on $\partial B_4(P)$ with the vacuum spinors, V. Anholonomic *changes* $\triangle\triangle\varphi^\alpha(V)\sim G$ in the vacuum spinors due to *sources* are what impart *curvature* $\triangle\triangle\varphi^\alpha(P)$ at the worldtube boundary $\partial B_4(P)$), via the discrete version of the boundary integral in (36).

Mass scatterings at the worldtube boundary account not only for the curvature of the worldtube $B_4(P)$, but also for the *annihilation* of P and *creation*

of intermediate particles by recombination of its spinor components with each other and with the vacuum spinors.

Each particle is composed of J *chiral pairs* of L- and R-chirality spinors: leptons of 1, mesons of 2, and hadrons of 3 pairs, respectively [4]. Their reactions are "crossover" exchanges of spinors from the ingoing to outgoing sets of particles—and with the $(4-J)$ remaining pairs of *vacuum* spinors that make the spacetime tetrads (17), metric tensor (16), and the effective vector potentials (9) and fields in (36). Particle propagators, like the Dirac propagator [8], thus include creation of intermediate particles and their return to the vacuum "sea". In this sense, the NM model is innately quantum mechanical, and does not need to be "quantized".

But wait! There are two basically different recipes, representing different underlying physical processes, for computing the "sum over null zig zags" in the Dirac propagator [8]!

R1) **The "sum over histories".** Create an ensemble Σ_C of singular *Spin*^{c}-4 complexes $\langle \Sigma_C \rangle$ with the same set of topological charges and particle trajectories, C, as our classical *Spin*^{c}*-4 complexes* and with isomorphic singular homologies. Each Σ_C represents a different microscopic history of mass scatterings and intermediate annhialation/creation events compatible with the classical history, C. *Now average over the ensemble of all such* Σ_C *to get quantum mechanical expectation values of observables*—just as we average over all microstates to get expectations values in statistical mechanics. These expectation values, $\langle \psi \rangle (p)$, evolve according to the Dirac propagator.

R2) **The stochastic process.** Create a random walk of the (lightlike) rays of matter spinors, ψ, and their mass-scatterings vertices, p, with the remaining *vacuum spinors, treated as random fields,* with mean distributions (18), and variances $|\triangle \boldsymbol{\varphi}|\,|\triangle \theta| = \frac{\hbar}{2}$. The resulting probability vector $\psi\,(p)$ for the matter fields evolves by the Dirac propagator.

In R1), there are "many worlds" that exist simultaneously. In R2), there is only one world—the *real Spin*^{c}-4 complex $\sum$—but we cannot know enough information about the vacuum spinors and their fluctuations to distinguish it from the other members Σ_C of its ensemble. Both remain valid interpretations of NM model, as they are for standard quantum mechanics, and even for statistical mechanics!

Suffice it to say here that a realistic model, with one real *Spin*^{c}-4 complex $\sum$ is not precluded, because the NM model is *nonlocal. Half* of the spinors incident on every vertex $p \in \sum$ in the *Spin*^{c}-4 complex—the conjugate-analytic spinors $\psi\,(\overline{z})$—propagate *backwards* in cosmic time, T, i.e. in the *opposite* direction to cosmic expansion. These carry only slightly less energy than the forward-propagating, analytic spinors $\psi\,(z)$. Macroscopically [18], the net result is that our Friedmann 3-simplex $\Sigma_3 \subset \sum$ expands much more slowly than the speed of light. Microscopically, the result R of a measurement may propagate *backwards* in T to become one of the phase increments $\triangle \zeta_R$ incident on p. R would thus

"bootstrap" cycles in $\sum$ connecting p and R—self-consistent causal cycles that are temporally *nonlocal.* "Paradoxical" cycles (like killing your own grandfather) would simply *not* boot-strap, and therefore not exist in the *real* $Spin^c$-4 complex. Contrapositively, all the *existing* vertices p and 1, 2, 3, and 4 cycles γ_J must be embedded self-consistentlly in the *real* $Spin^c$-4 complex—our whole world, how it was made, and what we make of it.

Any attempt to isolate a local simplex of $\sum$, and give a deterministic recipe for its T evolution, necessarily ignores links with other local simplices on its boundary, and with the global, but stochastic vacuum spinor fields. The best we can do is to average over the ensemble Σ_C of simplices with every possible configuration of boundary fields and vacuum spinors—and so our theory can only predict ensemble averages.

9 Conclusion

The vacuum spinors are what produce the inertial mass of a bispinor particle because mass scatterings with the vacuum fields are what confine chiral pairs of matter spinors to a timelike worldtube, B_4. Inside B_4, the matter spinors "zag" backwards in cosmic time T *almost* as often as they "zig" forward. Macroscopically, the timelike flux of the *spinfluid* decreases away from the boundary ∂B_4 of a source. This creates a "curl" K_{0r} in the vacuum flow that causes the worldtubes of test particles to curve, i.e to accelerate towards the source.

Spinfluid models thus give a *mechanism* for gravitation. Qualitatively,

i) vorticity arises on the boundary of each energy-momentum current, $*T$.

ii) rotation $\mathcal{R}$ of the spin fluid flow falls with distance from the source current. Its moment $*G$ at the boundary of the worldtube of a test current causes this worldtube to bend towards the source. Thus,

iii) neighboring centrifugal currents (masses) present *opposite* radiotemporal vorticities G_{or} to each other's worldtube boundaries—and therefore *attract* (or advect, like hydrodynamic vortices [11]).

Quantitatively, the gravitational constant is determined by the same balance between outward flow and boundary curvature that determines the equilibrium radius of the Friedmann universe. The power of spinfluid models to determine some constants of nature also makes them *falsifiable.* For example, relations (29) and (47) give the value

$$\kappa m_e^2 = \frac{\gamma^{-2}}{8}(T) \tag{60}$$

for the dimensionless constant that measures the ratio of gravitational to electromagnetic forces between electrons on our dilated Friedmann 3-brane, $\mathbb{S}_3(a)$. The predicted value of (60) would match the value of 10^{-39} observed today with

a dilation factor of $\gamma \sim 10^{20}$ and predicts that this ratio should *decrease* with cosmic expansion!

At $T = 0$, on $\mathbb{S}_3(a_\#)$, gravitational and electromagnetic forces have the same order of magnitude, because they have *not yet separated*. It is *cosmic expansion*—the breaking of T–reflection symmetry—that divides the forces into electroweak (PT-antisymmetric) and *gravitostrong* (PT-symmetric) sectors [2].

Microscopically, the action S_g is a sum over mass scatterings between J chiral pairs of matter spinors and $(4 - J)$ vacuum pairs plus the action of the perturbed vacuum spinor fields. Together, these make up a *Spin*^{c}-4 complex. The statistical mechanics in "imaginary time" $T \equiv y^0$ of all null *Spin*^{c}-4 complexes Σ_C compatible with a given set C of classical particle trajectories, masses, and charges is *quantum gravity.*

I don't know how our NM model compares with other theories of quantum gravity. There is a crucial experimental test, however. The NM model predicts a new effect in *supervortical* regimes: that Lens-Thirring fields should polarize weak decays, producing *left-helicity* jets about the rotation axis [2], [16]. Perhaps such axial jets could be measured in terrestrial laboratories or in astronomical observations.

More significant than the prediction of new effects of the derivation of fundamental constants and selection rules or are the qualitative features of the class of *quantum spinfluid* models that enable them to reconcile quantum mechanics and general relativity. These are

1. A Lagrangian density with *no* free parameters that is a *natural* 4 form—i.e. invariant under the group of passive spin isometries in curved spacetime, including symplectomorphisms.

2. An action that *includes* a bounded vacuum energy which depends on the radius $a(t)$ of the Friedmann solution. This breaks dilation invariance and sets all length and mass scales. It includes a repulsive term at high densities that prevents collapse to a singularity.

3. Values for the standard coupling constants that are either "frozen in" by the history of dynamical symmetry breaking, or that depend on cosmic time T through the radius $a(T)$.

4. Effective electroweak, strong, and gravitational *field* actions, along with minimal coupling through spin connections in the covariant derivatives.

5. Fields which are sourced in localized currents with *topologically quantized charges.* These charges parallel the electric charges, masses, spins, and lepton or baryon numbers of the observed families of particles.

6. A *mechanism* for gravitation derived from the *same* nonlinear coupling to the vacuum spinors that creates the *inertial masses* of particles.

7. Quantum propagators, including intermediate creation and annihilation operators, which are *derived* from the statistical mechanics of null zig-

zags of the (lightlike) spinor fields that weave the (timelike) worldtubes of massive particles and the (spacelike) fabric that connects them.

The NM model is the minimal model with these features, because:

a) It takes the intersection of 4 null cones to determine a point on $\mathbb{M}$.

b) Each nullcone is generated, via S^{-1} of (15), by the product (9) of 1 cospinor and 1 spinor differential. There must be **8** spinor fields in all; **4** PTC-equivalent pairs.

c) For *symplectic* invariance, each term in $\mathcal{L}_g$ must contain 4 spinors Ψ^I and 4 differentials $\mathbf{d}\Psi_I$. The symplectic forms are the 4 Maurer-Cartan 1 forms $\Psi^I \mathbf{d}\Psi_I = \Omega_I$. Their integrals are quantized over 1-cycles.

d) Under PTC symmetry, $\mathcal{L}_g$ reduces to the Maurer-Cartan (MC) 4 form. Its action S_g measures the covering number of the compactified internal group $U(1) \times SU(2)$ over the compactified spacetime manifold $\mathbb{M}_{\#}$. S_g is a topological invariant; it comes in integral units.

e) Since the vacuum energy has a topological upper bound of $16\pi^3 W$ [3], with W an integer that does not change with refinement of the lattice, there are no built-in divergences, and we don't have to worry about unbounded vacuum energies rolling up our space to a point. There is no need for renormalization.

f) The NM model admits *nonperturbative* solutions [3], [6] in the *superdense* regimes inside collapsed objects like neutron stars, black holes, and the early universe. These solutions are regularized by the resistance (26) of the topologically nontrivial vacuum to compression to a point [3]—a "pressure" of the vacuum spinors that contributes no additional energy-momentum (see Appendix).

The vacuum energy—or *dark energy*—is simply the energy of the homogeneous distribution of vacuum spinor fields $\left\{\hat{\psi}_I, \hat{\psi}^I\right\}$, on which the gauge fields ride like waves on the surface of an ocean. Material particles are the *caustics* of these vacuum spin waves [3], [4]—topological dislocations with quantized charges.

We have found some classical soliton solutions [24] for the unified field action $\mathcal{L}_g$ with topologically quantized charges [3]. These exemplify the inner solutions of Section 4. A stratification lemma [4] classifies the varieties of inner solutions that can exist, and their allowed reactions. These seem to parallel the observed particles and reactions, with *no exceptions*[4] noted so far.

[4]When I presented the NM model at CERN in July of 2000, T.T. Wu asked, "Can you account for neutral pion decay?" Wu's question was the gateway for me into spinor exchanges with the vacuum sea. It then became clear how even the simplest reactions and interactions, like photon emission and absorption, recruit spinors from the vacuum and then return them [4].

The above results all support our picture of the $Spin^c$-4 complex of global vacuum spinors, matter spinors, and their mass scatterings *as* the microscopic reality. Spacetime, gauge fields, and the observed varieties of matter fields and their reactions, including gravitation, *emerge* from dynamical stratification [4] of the $Spin^c$-4 complex as 1, 2, 3, and 4 chiral pairs of spinor fields break away from the PTC-symmetric vacuum distribution.

10 Acknowledgments and Dedication

Thanks go to Jaime Keller [20], who first understood and explained to me the advantages of the chiral $gl\,(2,\mathbb{C})$ presentation over the twistor presentation of the conformal group for massive bispinor particles. I am very grateful to T.T. Wu for listening carefully to my presentation of the NM model at CERN in July, 2000—and for setting me the riddle of neutral pion decay. Thanks go to Astri Kleppe for first helping me to draw the correspondence of chiral bispinor pairs and exchanges among the **8** spinors to real particles and reactions. Eternal thanks go to Elaine and Mikaela Cohen for keeping my feet grounded on this Earth.

This paper is dedicated to the memory of my mother, Florence Channock Cohen, whose spirit returned from timelike weave of matter into eternal light on January 19, 2003.

11 Appendix: Conformal Symmetry Breaking and Particle Nucleation

Time evolution preserves topological invariants. Like kinks and knots in a hose as it is pulled tight, topological defects in the spinfluid localize with cosmic expansion, and they acquire *mass*.

Like the residues of complex scalar fields, our action integral has a nested set of complex *quaternionic* (Clifford) *residues* [3]: integrals over codimension-J *singular loci* $D^J \subset \mathbb{C}_4 \subset T^*\mathbb{M}$ in phase space, where the PTC-symmetric geometrical optics ansatz (1) breaks down. On cycles $\gamma_{4-J} \equiv *D^J$, $J = 1, 2,$ 3, or 4 chiral pairs $\widetilde{\Omega}$ "de-loop" from their global vacuum distributions (18) and appear as localized *matter spinors*, quantized over closed $(4-J)$-branes. They thus acquire *masses* and topological charges. The action of the W chiral pairs of *vacuum spinors* topologically trapped on $\widehat{\mathbb{M}}$ integrates to a constant, $-16\pi^3 W$, because their intensities $|\Psi_I|^2$ scale as γ^{-4}, and the volume element scales as γ^4. However, the action of each *matter-spinor* pair topologically trapped on a codimension-J cycle D^J scales as γ^J when integrated over the J orthogonal directions. Since the matter spinors in supports $\gamma_{4-J} \equiv *D^J$ meld continuously into the surrounding vacuum $\widehat{\mathbb{M}}$, their supports γ_1, γ_2, γ_3, and γ_4 become J =1, 2, 3, and 4-cycles—over which their phase differentials (Lie-algebra-valued forms) $\widetilde{\Omega}^1$, $\widetilde{\Omega}^2$, $\widetilde{\Omega}^3$, and $\widetilde{\Omega}^4$ are *quantized* [3]—and so their integrals do not change

with γ:

$$\begin{aligned} \textstyle\int_{\widehat{\mathbb{M}}} \widehat{\Omega}^4 &= 16\pi^3 W; \\ \sum_{D^1} \textstyle\int_{S_3\times\gamma_1} \widehat{\Omega}^3 \wedge \widetilde{\Omega}^1 &= i8\pi^2\gamma \sum_{D^1} i2\pi m \equiv -16\pi^3 M\gamma \\ \sum_{D^2} \textstyle\int_{S_2\times\gamma_2} \widehat{\Omega}^2 \wedge \widetilde{\Omega}^2 &= 4\pi\gamma^2 \sum_{D^2} 4\pi q \equiv 16\pi^3 Q\gamma^2 \\ \sum_{D^3} \textstyle\int_{S_1\times\gamma_3} \widehat{\Omega}^1 \wedge \widetilde{\Omega}^3 &= i2\pi\gamma^3 \sum_{D^3} i8\pi^2 B \equiv -16\pi^3 N\gamma^3. \end{aligned} \tag{61}$$

Note that the topological charges of the matter spinors become quantized over the *dual* cycles γ_{4-J} spanned by the $4-J$ pairs of *vacuum spinors,* $\widehat{\Omega}^{4-J}$ [3].

Elsewhere [3], [4] we have identified

M as the number of *leptons* (weighted by charge),
Q as the total *charge,* and
N as the number of *hadrons* (weighted by charge)

contained in $\mathbb{M}$.

The net effect of (61) is to produce a self-potential well $V(\gamma)$ in which the scale factor $\gamma(t)$ moves in Minkowsky time, t.

Both the vacuum and matter spinors acquire some *kinetic energy* relative to our expanding 3-brane $\mathbb{S}_3(T)$, as it sweeps through their rest frames. Thus the Friedmann action $S\left(\gamma, \dot{\gamma}\right) \equiv T\left(\dot{\gamma}\right) - V(\gamma)$ contains a kinetic energy term $T\left(\dot{\gamma}\right)$ in the expansion rate $\dot{\gamma}$. To obtain $T\left(\dot{\gamma}\right)$, we re-express our action integral in a Lorenz frame comoving with the Friedmann flow:

$$\begin{aligned} \left(E^0, E^1, E^2, E^3\right) &= \gamma^4\beta\left(\mathbf{d}y^0, e^1, e^2, e^3\right); \\ \beta = \left(1 + a_\#^2\dot{\gamma}^2\right)^{\frac{1}{2}} &\Longrightarrow \mathbf{d}^4V' = \left(1 + a_\#^2\dot{\gamma}^2\right)^2 \gamma^4\mathbf{d}^4v \equiv \gamma'^4\mathbf{d}^4v, \end{aligned} \tag{62}$$

where $a_\#\dot{\gamma} \equiv \dot{y}^0$ is the expansion rate of the logradius (cosmic time) $T \equiv y^0$ with Minkowsky time (arctime) $t \equiv x^0$.

We then integrate over $\mathbb{M}' \equiv (T, \mathbb{S}_3(T)) \subset \mathbb{C}_4$, our expanding hyperspherical shell thickened up by cosmic ("imaginary") time T to obtain the action $S\left(\gamma, \dot{\gamma}\right)$ it contains. We only need to keep track of the *scalar* (σ_0) terms in $\mathcal{L}_g$—because only Clifford *scalars* contribute to the *Trace.*

Next, we expand the scalar (σ_0) terms in S by orders in the net conformal weight, γ^p. Altogether, we get terms with conformal weights $p = [-4, -3, -1, 0]$ in our effective Lagrangian $\widehat{\mathcal{L}}_g(\psi, \mathbf{d}\psi)$ for our Friedmann vacuum $\widehat{\mathbb{M}}$ with its embedded topological twists: spacetime, with a "foam" of singular loci—the massive particles. These integrate to terms with conformal weights, γ^p: $p = [0, 1, 2, 3, 4]$. As γ grows, heavier particles begin to dominate the action.

But, what cuts out our expanding space $\mathbb{M}' = (T, S_3(T)) \subset \mathbb{C}_4$ in the ambient phase space is the *bulk neutrality condition,* $Q = O$ in (61). This leaves us with terms in γ and γ^3 in our potential energy $V(\gamma)$. Meanwhile, the

$a_{\#}\dot{\gamma} = \dot{y}^{0}$ term in the volume expansion ratio in the *Lorenz* frame comoving with the Friedmann flow provides the kinetic energy term $T\left(\dot{\gamma}\right)$, giving

$$S\left(\gamma, \dot{\gamma}\right) = 16\pi^3 \left(1 + a_{\#}^2 \dot{\gamma}^2\right)^{\frac{1}{2}} \left[W - M\gamma - N\gamma^3\right] \tag{63}$$

for the action contained in $\mathbb{M}'$. Defining

$$\tau \equiv \left(1 + a_{\#}^2 \dot{\gamma}^2\right)^{\frac{1}{2}} t$$

as the comoving time increment, then varying $S\left(\gamma, \dot{\gamma}'\right)$ with respect to γ and $\gamma' \equiv \partial_\tau \gamma$ gives

$$a_{\#}^2 \gamma'' = \partial_\gamma \ln\left[W - M\gamma - N\gamma^3\right] \equiv -\partial\gamma V\left(\gamma\right) \tag{64}$$

as the differential equation for the scale factor, γ.

The radius $a\left(\tau\right) \equiv \gamma\left(\tau\right) a_{\#}$ of our Friedmann 3-brane moves in a self-potential well

$$V\left(\gamma\right) \equiv -\ln\left[W - M\gamma - N\gamma^3\right]$$

with an equilibrium value of

$$\gamma_{\#} = \left(\frac{-M}{3N}\right)^{\frac{1}{2}}. \tag{65}$$

$\gamma_{\#}$ is *unstable* (inflationary) for $M > 0$, $N < 0$, and *stable* (oscillatory) for $M < 0$, $N > 0$. "Natural selection" seems to prefer a universe with negatively-charged leptons and positive hadrons!

In the NM model [2], [3], [4] a *hadron* is a bound configuration of 3 out of the 4 chiral pairs of spinor fields; a lepton is one bound bispinor pair. Since a homotopy may split one codimension-3 singular locus D^3 into a tensor product of three codimension-1 locii D^1, the ratio $M = 3N$ is *generic* (homotopy invariant). This gives $\gamma_{\#} = 1$ in (65). The compactification radius $a_{\#}$, which entered *kinematically* [25] as the quantization length (4) for global $u\left(1\right) \times su\left(2\right)$ phase gradients $d\theta^\alpha \sigma_\alpha$ in (4), turns out to be the *dynamical* equilibrium radius of our Friedmann 3-brane.

Interestingly enough, $2a_{\#}$ also turns out to be the *Compton wavelength of the electron,* $2a_{\#} = m_e^{-1}$, as we show in Section 4 of the text.

Note that to prevent the argument for the ln in $V\left(\gamma\right)$ from going negative, W must exceed

$$W_{\#} \equiv \frac{-2M}{3}\left(\frac{-M}{3N}\right)^{\frac{1}{2}}. \tag{66}$$

$W_{\#}$ is the lower bound on the number of chiral pairs of vacuum spinors needed to prevent a singularity in the time evolution of the radius $a\left(\tau\right)$ of the Friedmann

solution. The positive sign on W indicates *left helicity* of the vacuum spinors [25]; i.e. $su(2)_L$ twist along null rays. Thus we detect left-helicity bispinors like $v_e \equiv (\xi_+ \oplus \eta_-)$; not $\overline{v}_\ell \equiv (\xi_- \oplus \eta_+)$, in the vacuum spinor flux.

In a universe composed of *matter* ($M < 0$, $N > 0$) rather than antimatter, a stable equilibrium $\gamma_\#$ is produced when the aggregative (gravitostrong) force between massive hadrons (N) is balanced by the quantum-mechanical preference of leptons (M) for *delocalization.* This balance lives in the domain of electroweak-gravitostrong unification with a *regular,* topologically-nontrivial distribution of *vacuum spinors*; it *cannot* be captured by ad-hoc cutting and pasting of quantum field theory and general relativity.

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